**NEHRU COLLEGE OF ENGINEERING AND RESEARCH CENTRE**
**(NAAC Accredited)**
(Approved by AICTE, Affiliated to APJ Abdul Kalam Technological University, Kerala)

**DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING**

# *COURSE MATERIAL*

# *EE 306 POWER SYSTEM ANALYSIS*

## VISION OF THE INSTITUTION

To mould our youngsters into Millennium Leaders not only in Technological and Scientific Fields but also to nurture and strengthen the innate goodness and human nature in them, to equip them to face the future challenges in technological break troughs and information explosions and deliver the bounties of frontier knowledge for the benefit of humankind in general and the down-trodden and underprivileged in particular as envisaged by our great Prime Minister Pandit Jawaharlal Nehru

## MISSION OF THE INSTITUTION

To build a strong Centre of Excellence in Learning and Research in Engineering and Frontier Technology, to facilitate students to learn and imbibe discipline, culture and spirituality, besides encouraging them to assimilate the latest technological knowhow and to render a helping hand to the under privileged, thereby acquiring happiness and imparting the same to others without any reservation whatsoever and to facilitate the College to emerge into a magnificent and mighty launching pad to turn out technological giants, dedicated research scientists and intellectual leaders of the society who could prepare the country for a quantum jump in all fields of Science and Technology

## ABOUT DEPARTMENT

- Established in: 2004
- Courses offered : B.Tech in Electrical and Electronics Engineering

  M.Tech in Energy Systems
- Approved by AICTE New Delhi and Accredited by NAAC
- Affiliated to the A P J Abdul Kalam Technological University.

## DEPARTMENT VISION

To excel in technical education and research in the field of Electrical & Electronics Engineering by imparting innovative engineering theories, concepts and practices to improve the production and utilization of power and energy for the betterment of the Nation.

## DEPARTMENT MISSION

- To offer quality education in Electrical and Electronics Engineering and prepare the students for professional career and higher studies and to make students socially responsible
- To create research collaboration with industries for gaining knowledge about real-time problems.

### PROGRAM OUTCOMES (POS)

**Engineering Graduates will be able to:**

1. **Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of

mathematics, natural sciences, and engineering sciences.

3. **Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and team work**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one’s own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. **Life-long learning**: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

**PROGRAM SPECIFIC OUTCOMES (PSO)**

**PSO1**: Ability to Formulate the various power system components under normal and abnormal conditions

**PSO2**: Ability to learn and solve the problems related to load flow studies under normal and abnormal conditions

**PSO3**: Ability to inculcate the Knowledge for analyzing different stability criteria and its solution for fault clearance

## Note: H-Highly correlated=3, M-Medium correlated=2, L-Less correlated=1

| CO'S | PO1 | PO2 | PO3 | PO4 | PO5 | PO6 | PO7 | PO8 | PO9 | PO10 | PO11 | PO12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C306.1 | 2 | 1 | | | | | | | | | | |
| C306.2 | 3 | 1 | | | | | | | | | | |
| C306.3 | 3 | 1 | | | | | | | | | | |
| C306.4 | 3 | | | | | | | | | | | |
| C306.5 | 3 | | | | 1 | | | | | | | 2 |
| C306.6 | 3 | | | | 2 | | | | | | | 2 |
| C306 | 2.83 | 0.5 | | | 0.5 | | | | | | | 0.66 |

| SUBJECT CODE: EE306 | |
|---|---|
| COURSE OUTCOMES | |
| C306.1 | To enable the students to analyse power systems under normal and abnormal conditions |
| C306.2 | To understand the need for load flow analysis and different methods |
| C306.3 | To understand power system modeling |
| C306.4 | To analyse the single and double area function for automatic system control |
| C306.5 | Identify different economic operation conditions for given load |
| C306.6 | To understand the need for stability studies and their analysis |

| CO'S | PSO1 | PSO2 | PSO3 |
|---|---|---|---|
| C306.1 | 3 | 3 | 3 |
| C306.2 | | 3 | 3 |

| | | | |
|---|---|---|---|
| C306.3 | 3 | | |
| C306.4 | | 3 | 3 |
| C306.5 | 3 | | |
| C306.6 | | | 2 |
| C306 | 3 | 3 | 2.75 |

| Course code | Course Name | L-T-P - Credits | Year of Introduction |
|---|---|---|---|
| **EE306** | **POWER SYSTEM ANALYSIS** | **3-0-0-3** | **2016** |

**Prerequisite: Nil**

**Course Objectives**

- To enable the students to analyse power systems under normal and abnormal conditions.
- To understand the need for load flow analysis and different methods
- To understand power system modeling
- To understand the need for stability studies and their analysis

**Syllabus**

Per unit quantities - modeling of power system components - methods of analyzing faults in symmetrical and unsymmetrical case - load flow studies - Automatic Generation Control - Automatic voltage control – Economic load dispatch - Unit commitment - Power system stability - Solution of swing equation - Methods of improving stability limits

**Expected outcome** .

The students will be able to:

i. Analyse power systems under normal and abnormal conditions.
ii. Carry out load flow studies under normal and abnormal conditions

**References:**


1. Cotton H. and H. Barber, Transmission & Distribution of Electrical Energy, 3/e, Hodder and Stoughton, 1978.
2. Gupta B. R., *Power System Analysis and Design*, S. Chand, New Delhi, 2006.
3. Gupta J.B., Transmission *& Distribution of Electrical Power*, S.K. Kataria & Sons, 2009.
4. Hadi Saadat, *Power System Analysis*, 2/e, McGraw Hill, 2002.
5. Kothari D. P. and I. J. Nagrath, *Modern Power System Analysis*, 2/e, TMH, 2009.
6. Kundur P., *Power system Stability and Control*, McGraw Hill, 199
7. Soni, M.L., P. V. Gupta and U. S. Bhatnagar, *A Course in Electrical Power*, Dhanpat Rai & Sons, New Delhi, 1984.
8. Stevenson W. D., *Elements of Power System Analysis*, 4/e, McGraw Hill, 1982.
9. Uppal S. L. and S. Rao, *Electrical Power Systems*, Khanna Publishers, 2009.
10. Wadhwa C. L., *Electrical Power Systems*, 33/e, New Age International, 2004.
11. Weedy B. M., B. J. Cory, N. Jenkins, J. B. Ekanayake and G. Strbac, *Electric Power System*, John Wiley & Sons, 2012.

| Course Plan | | | |
|---|---|---|---|
| **Module** | **Contents** | **Hours** | **Sem. Exam Marks** |
| I | Per unit quantities-single phase and three phase-selection of base quantities -advantages of per unit system –changing the base of per unit quantities-Simple problems. | 2 | 15% |
| | Modelling of power system components - single line diagram – per unit quantities. Symmetrical components- sequence impedances and sequence networks of generators, transformers and transmission lines. | 3 | |
| II | Methods of analyzing faults in symmetrical and unsymmetrical case- effects of faults - Power system faults - symmetrical faults - short circuit MVA - current limiting reactors- Unsymmetrical faults - single line to ground, line to line, double line to ground faults -consideration of prefault current-problems. | 8 | 15% |
| **FIRST INTERNAL EXAMINATION** | | | |
| III | Load flow studies – Introduction-types-network model formulation - formation of bus impedance and admittance matrix, Gauss-Siedel (two iterations), Newton-Raphson (Qualitative analysis only) and Fast Decoupled method (two iterations) - principle of DC load flow. | 8 | 15% |
| IV | Automatic Generation Control: Load frequency control: single area and two area systems - Automatic voltage control. | 6 | 15% |
| **SECOND INTERNAL EXAMINATION** | | | |


| | | | |
|---|---|---|---|
| III | Load flow studies – Introduction-types-network model formulation - formation of bus impedance and admittance matrix, Gauss-Siedel (two iterations), Newton-Raphson (Qualitative analysis only) and Fast Decoupled method (two iterations) - principle of DC load flow. | 8 | 15% |
| IV | Automatic Generation Control: Load frequency control: single area and two area systems - Automatic voltage control. | 6 | 15% |
| **SECOND INTERNAL EXAMINATION** | | | |


| | | | |
|---|---|---|---|
| V | Economic Operation - Distribution of load between units within a plant - transmission loss as a function of plant generation - distribution of load between plants - Method of computing penalty factors and loss coefficients. | 5 | 20% |
| | Unit commitment: Introduction — Constraints on unit commitments: Spinning reserve, Thermal unit constraints-Hydro constraints. - | 2 | |
| VI | Power system stability - steady state, dynamic and transient stability-power angle curve-steady state stability limit | 3 | 20% |
| | Mechanics of angular motion-Swing equation – Solution of swing equation - Point by Point method - RK method - Equal area criterion application - Methods of improving stability limits. | 5 | |
| **END SEMESTER EXAM** | | | |

QUESTION PAPER PATTERN:

# MODULE NOTES

I

Per unit quantities & percent quantities

The per unit value of any quantity is defined as the ratio of the actual value of the quantity to the base value expressed as a decimal (fraction). The ratio in percent is 100 times the value in per unit. The base value is an arbitrary chosen value of the quantity.

$$\text{Per unit value} = \frac{\text{Actual value}}{\text{Base value}}$$

$$\text{\% per unit value} = \frac{\text{Actual value}}{\text{Base value}} \times 100$$

→ The power system requires the base values of four quantities and they are voltage, power, current and impedance. Selection of base values for any two of them determines the base values of the remaining two.

→ The various components of power systems have their voltages, power, current and impedance ratings in kV, kVA, kA and Ω respectively

→ Usually base mega volt ampere and base voltage in kV are the quantities selected as base

(i) units of both actual & base value should be same

(ii) Base value should be a real value.

Advantages

1) The per unit impedance referred to either side of a single phase transformer is the same

2) The per unit impedance referred to either side of 3φ transformer is the same regardless of the 3φ connections whether they are Y-Y, Δ-Δ, Δ-Y

3) The chance of confusion between line and phase quantities in a 3φ balanced system is greatly reduced

4) The manufacturers usually provide the impedance values in pu

5) The computational effort in power systems is very much reduced with the use of per unit quantities). Usually the per unit quantities being of the order of unity or less, can easily be handled with a computer. Manual calculations are also simplified.

$$\text{Base impedance}, \Omega = \frac{\text{Base voltage}}{\text{Base current}}$$

$$= \frac{\text{Base } kV_{LL} / \sqrt{3}}{\text{Base } MVA_{\cancel{3\phi}} / \cancel{\sqrt{3}} \times \text{Base } kV_{LL}}$$

$P = VI$

$MVA = kV \times kA$

$current_b = \frac{MVA_b}{kV_b}$

$\therefore Z_b = \frac{V_b}{Curr_b}$

$= \frac{kV_b}{MVA_b / kV_b}$

$$\text{Base impedance} = \frac{(\text{Base } kV_{LL})^2}{\text{Base } MVA_{3\phi}}$$

Note: The formula to calculate base impedance is the same irrespective of single phase or three phase systems.

Changing the base of per unit quantities

→ If the values given are already in the p.u values referred by their own ratings, then to convert them to the selected base values,

$$Z_{p.u}^{(new)} = Z_{p.u}^{(old)} \times \left( \frac{kV_{base}^{(old)}}{kV_{base}^{(new)}} \right)^2 \times \frac{MVA_{base}^{(new)}}{MVA_{base}^{(old)}}$$

Proof

$$\text{P.u impedance} = \frac{\text{Actual impedance}}{\text{Base impedance}}$$

$$Z_{p.u}^{(old)} = \frac{Z_{actual}}{\frac{(kV_b^{(old)})^2}{MVA_b^{(old)}}}$$

$$= \frac{Z_{actual}}{(kV_b^{(old)})^2} \times MVA_b^{(old)} \quad \text{——— (1)}$$

$\text{III}^{ly}$ when referred to new base values,

$$Z_{p.u}^{(new)} = \frac{Z_{actual}}{(kV_b^{new})^2} \times MVA_b^{new} \quad \text{——— (2)}$$

## Percent quantities

The per unit value of any quantity is defined as

$$\frac{\text{the actual value in any unit}}{\text{the base (reference) value in the same unit}} \times 100$$

* Percent value = per unit value × 100

Problem

1) For a base voltage of 11 kV and base MVA of 1000, convert 2 Ω into per unit

$$Z_{pu} = \frac{Z_{actual}}{Z_{base}}$$

Given

$Z_{actual} = 2\,\Omega$

$$Z_{base} = \frac{(\text{Base kV})^2}{\text{Base MVA}}$$

$$= \frac{11^2}{1000 \times 10^{-3}} = 121\,\Omega$$

$$\therefore Z_{p.u} = \frac{Z_{actual}}{Z_{base}}$$

$$= \frac{2}{121} = 0.0165 \text{ pu}$$

note 2 In case of 3 identical single phase transformers is given instead of 3 phase transformer, multiply the KVA rating of single phase transformer by three and star side line voltage rating has to be multiplied by $\sqrt{3}$ and continue the problem as stated above.

1) A $3\phi$ generator with rating 1000 KVA, 33kV has its armature resistance and synchronous reactance as $20\,\Omega/ph$ and $70\,\Omega/ph$. Calculate p.u impedance of the generator

Sol

The gr ratings are chosen as base kV and base KVA

$$\text{Base imp } Z_b = \frac{kV_b^2}{MVA_b} = \frac{33^2}{1000/1000} = 1089\,\Omega$$

$$\text{Actual imp}/ph = Z = 20+j70 \ \Omega/ph.$$

$$\therefore Z_{pu} = \frac{Z}{Z_b} = \frac{20+j70}{1089} = 0.018+j0.064 \text{ pu}$$

① (a) A generator is rated 500 MVA, 22 kV. Its Y connected winding has a reactance of 1.1 p.u. Find the ohmic value of the reactance of winding

(b) If the ~~same~~ given generator is working in a circuit for which the bases are specified as 100 MVA, 20 kV. Then find the p.u value of reactance of generator winding on the specified base.

Solution

(a) The generator p.u reactance will be specified by taking its rating as base values.

$\therefore\ kV_b = 22\ kV$

$MVA_b = 500\ MVA$

$$\text{Base impedance},\ Z_b = \frac{kV_b^2}{MVA_b} = \frac{22^2}{500} = 0.968\ \Omega$$

$$\text{per unit reactance},\ X_{pu} = \frac{\text{Actual reactance},\ \Omega}{\text{Base impedance},\ \Omega} = \frac{X}{Z_b}$$

$$\therefore\ \text{Actual reactance},\ X = X_{pu} \times Z_b$$

$$= 1.1 \times 0.968 = 1.0648\ \Omega/\text{phase}$$

(b) The formula used to convert the p.u reactance specified on a base value to another base is given below.

$$X_{pu}^{new} = X_{pu}^{old} \times \left(\frac{kV_b^{old}}{kV_b^{new}}\right)^2 \times \frac{MVA_b^{new}}{MVA_b^{old}}$$

$$= 1.1 \times \left(\frac{22}{20}\right)^2 \times \left(\frac{100}{500}\right)$$

$$= 0.2662\ \text{p.u}$$

New base values are, $kV_b^{new} = 20\ kV$, $MVA_b^{new} = 100\ MVA$

old base values are, $kV_b^{old} = 22\ kV$, $MVA_b^{old} = 500\ MVA$

Steady state :→ currents and voltages of the system have constant amplitude and frequency sinusoidal functions.

Transient state :→ The behaviour of the currents and voltages when they are changed from one state to another state.

→ During the different operating conditions such as normal, faulty and switching conditions, the currents and voltages vary enormously before reaching the steady state values.

## One line diagram (Single line diagram)

→ To draw the $1\phi$ equivalent ckt of sys

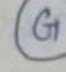
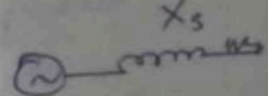
A single line diagram is diagrammatic representation of power system in which the components are represented by their symbols and the interconnections between them are shown by straight lines. Besides the symbols, the ratings and the impedances of the components are also marked on the single line diagram.

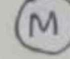
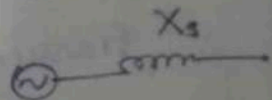
→ The purpose of the one line diagram is to supply in concise form the significant information about the system.

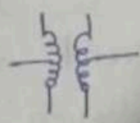
→ It is a convenient practical way of network representation rather than drawing the actual $3\phi$ diagram which may indeed be quite cumbersome & confusing for a practical size power network.

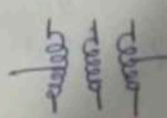
→ A balanced $3\phi$ system is always analysed on per phase basis by considering one of the three phase lines & neutral. Hence it is enough if we show one phase & neutral in the diagrammatic representation of power system. The diagram is further simplified by omitting the neutral and so the resultant diagram will be a single line diagram.

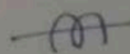
symbols used in single line diagrams

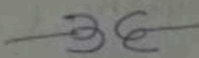
1) AC rotating machines - Generator →  (or) 

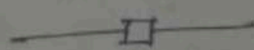

Motor →  (or) 


2) Two winding transformers →  (or) 

3) Three winding transformers →  (or) 

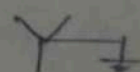
4) Current transformer → 

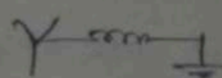
5) Potential transformer → 

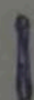
6) Power circuit breaker → 

7) Delta connection → $\Delta$

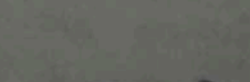
8) Star connection (neutral isolated) → Y

9) Star connection (neutral solidly grounded) → 

10) Star connection (Reactor earthed neutral) → 

11) Bus → 

12) OH line / cable (Tr. line) T (or) $\pi$ → ——

13) Isolator → 

14) Load (static) → ⟶

15) Lightning arrester $\longrightarrow$

16) Ammeter $\longrightarrow$ —(A)—

17) Voltmeter $\longrightarrow$ —(V)—

Generator : MVA , kV , subtransient reactance $X''$.

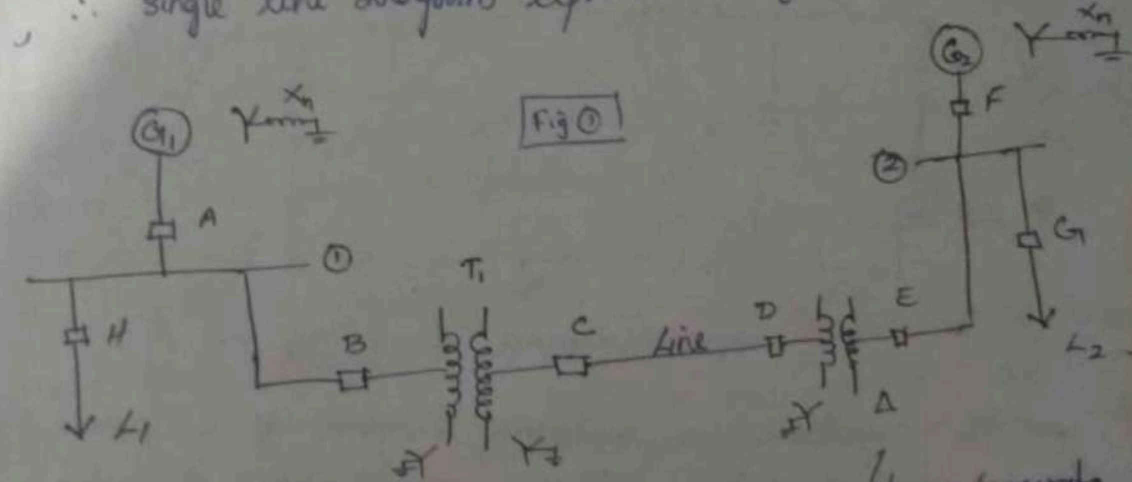
Transformer : $V_p / V_s$ , MVA , X

Load : P, Q (or) S , P.f

note: A $3\phi$ balanced system is always solved as a single phase circuit, $\therefore$ single line diagram represents only one phase of the system

eg;

Fig ①




→ circle represents one phase of a rotating machinery (here Generator 1 and 2) with the neutral is grounded through reactance $X_n$

→ Two winding transformer $T_1$ with both primary and secondary neutrals are solidly earthed. Two winding transformer $T_2$ with secondary (line side) neutral solidly earthed & the Py is delta connected.

→ Loads ($L_1$ and $L_2$) and circuit breaker (A to H) are represented shown as a small square box

→ Buses 1 and 2 are marked as thick bold lines.

## Impedance and reactance diagram

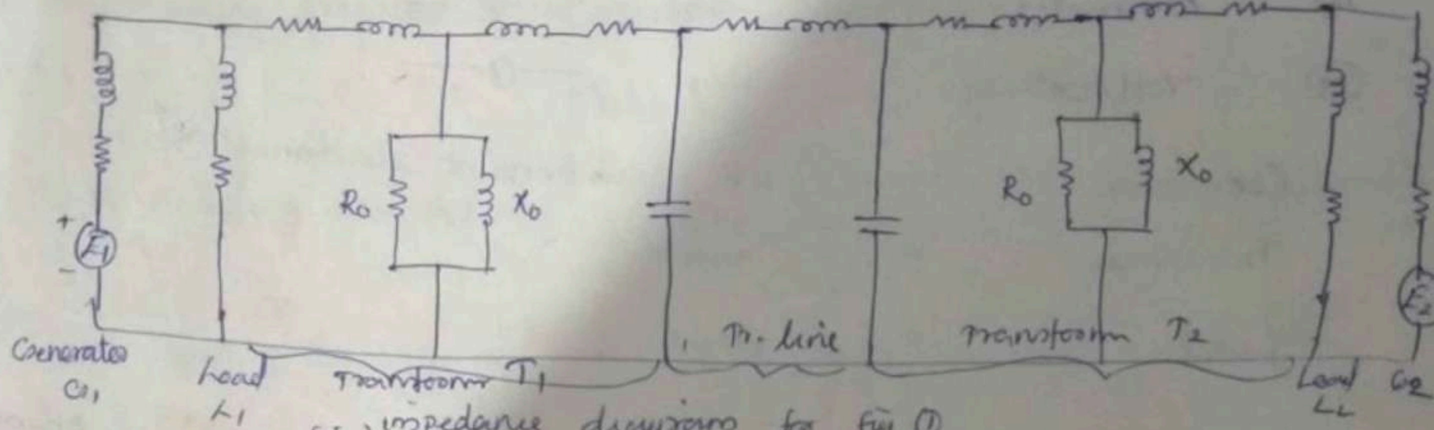



Fig: impedance diagram for fig ①

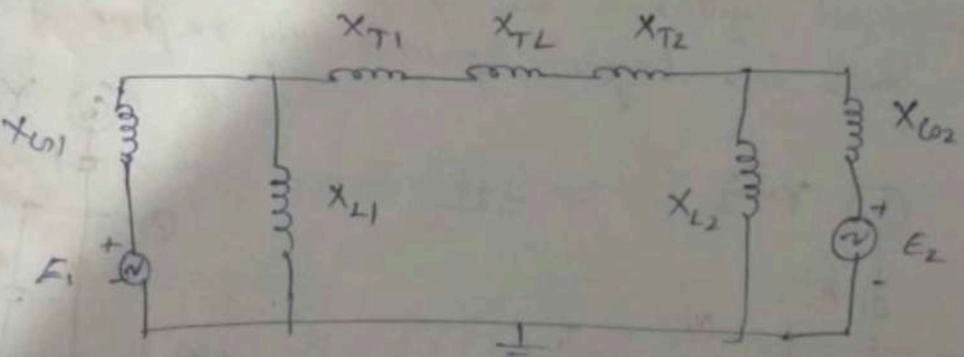



fig: Reactance diagram for fig ①

→ The impedance or reactance diagram of a power system is the equivalent circuit of the power system in which the various components of the power system are represented by their approximate or simplified equivalent circuits.

(i) Impedance diagram

The impedance diagram can be obtained from the single line diagram by replacing all the components of the power system by their single phase equivalent circuit.

assumptions

① The current limiting impedances connected btw the generator neutral & ground are neglected since under balanced conditions no current flows through neutral.

② Since the magnetizing currents of a transformer is very low when compared to load current the shunt branches in the equivalent circuit of the transformer can be neglected.

③ If the inductive reactance of a component is very high when compared to resistance then the resistance can be omitted, which introduces a little error in calculations

(ii) Reactance diagram

The reactance diagram can be obtained from impedance diagram if we omit R & all static loads, all resistances, shunt branches of transformer and capacitance of tr. lines in the impedance diagram

Assumptions

① The neutral to ground impedance of the generator is neglected for symmetrical faults

② Shunt branches in the equivalent circuits of transformer are neglected.

③ The resistances in the equivalent circuits of various components of the system are omitted

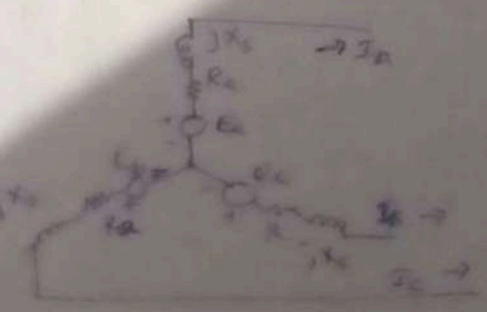
④ All static loads are neglected.

⑤ Induction motors are neglected in computing fault current a few cycles after the fault occurs, because the current contributed by an induction motor dies out very quickly after the IM is short circuited.

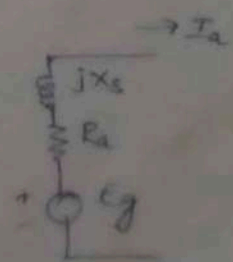
⑥ The capacitance of the tr. lines are neglected

Equivalent circuits of power system components

① 3 $\phi$ generator (alternator)




⟹




1φ equivalent circuit

(2) 3 generators are rated as follows.

Generator 1 : 100 MVA, 33 kV, reactance of 10 %

2 : 150 MVA, 32 kV, 8 %

3 : 110 MVA, 30 kV, 12 %

Determine the reactance of the generators corresponding to base values of 200 MVA and 35 kV.

Solution

Generator 1 : 100 MVA, 33 kV, $x'' = 0.1$

$$Z_{pu}^{(new)} = Z_{pu}^{(old)} \times \left(\frac{kV_b^{old}}{kV_b^{new}}\right)^2 \times \frac{MVA_b^{new}}{MVA_b^{old}}$$

$$x' = 0.1 \times \left(\frac{33}{35}\right)^2 \times \frac{200}{100} = 0.1778 \text{ p.u}$$

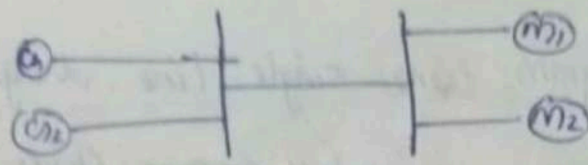
Generator 2 : 150 MVA, 32 kV, $x'' = 0.08$

$$x' = 0.08 \times \left(\frac{32}{35}\right)^2 \times \frac{200}{150} = 0.0892 \text{ pu}$$

Generator 3 : 110 MVA, 30 kV, $x'' = 0.12$

$$x' = 0.12 \times \left(\frac{30}{35}\right)^2 \times \frac{200}{110} = 0.1602 \text{ p.u}$$

(3) Two generators rated at 10 MVA, 13.2 kV and 15 MVA, 13.2 kV are connected in parallel to bus bar. They feed supply to two motors of input 8 MVA, and 12 MVA respectively. The operating voltage of motor is 12.5 kV. Assuming base quantities as 50 MVA and 13.8 kV, draw the reactance diagram. The percentage reactance for generators is 15 % and that for motors is 20 %





Generator 1 : 10 MVA, 13.2 kV, $x'' = 0.15$

$$Z_{pu}^{(new)} = x'' = 0.15 \times \left(\frac{13.2}{13.8}\right)^2 \times \frac{50}{10} = 0.6862 \text{ pu}$$

Generator 2 : 15 MVA, 13.2 kV, $x'' = 0.15$

$$x'' = 0.15 \times \left(\frac{13.2}{13.8}\right)^2 \times \frac{50}{15} = 0.4575 \text{ pu}$$

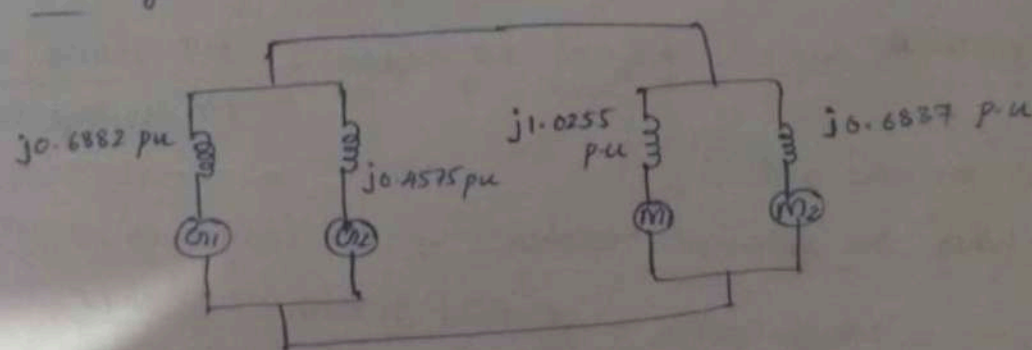
Motor 1 : 8 MVA, 12.5 kV, $x'' = 0.2$

$$x'' = 0.2 \times \left(\frac{12.5}{13.8}\right)^2 \times \frac{50}{8} = 1.0255 \text{ pu}$$

Motor 2 : 12 MVA, 12.5 kV, $x'' = 0.2$

$$x'' = 0.2 \times \left(\frac{12.5}{13.8}\right)^2 \times \frac{50}{12} = 0.6837 \text{ pu}$$

Reactance diagram





Base MVA will be same for the entire ckt

Base kV in each part depends on transformation ratio

Procedure to form reactance diagram from single line diagram

(i) Select a base kilo volt ampere or mega volt ampere ($kVA_b$ or $MVA_b$). The $kVA_b$ or $MVA_b$ will be same for all sections of the power system, ~~the~~ In case of three phase power system, the $kVA_b$ or $MVA_b$ is three phase power rating.

(ii) Select a base kilo volt ($kV_b$) for one section of power system. In case of three phase power system, the $kV_b$ is a line value. The various sections of power system works at different voltage levels and the voltage conversion is achieved by means of transformers. Hence the $kV_b$ of one section of power system should be converted to a $kV_b$ corresponding to another section using the transformer voltage ratio. In case of three phase transformer, line-to-line voltage ratio is used to transfer the $kV_b$ on one section to another section.

$$kV_b \text{ on LT section} = kV_b \text{ on HT section} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$kV_b \text{ on HT section} = kV_b \text{ on LT section} \times \frac{\text{HT voltage rating}}{\text{LT voltage rating}}$$

(iii) Find per unit values,

(a) When the specified reactance of the component is in ohms then

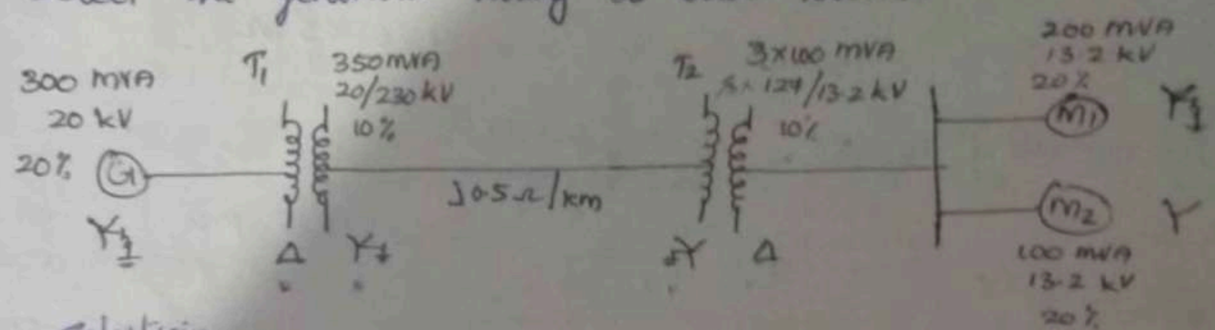
$$\underset{X_{pu}}{\text{p.u reactance}} = \frac{\text{Actual reactance in ohms}}{\text{Base impedance}}$$

(b) When the specified reactance of the component is in p.u. on the component rating as base values, then consider the component rating as old base values and selected base values as new base. Now the p.u reactance on new base can be calculated using the formula

$$X_{pu}^{new} = X_{pu}^{old} \times \left(\frac{kV_b^{old}}{kV_b^{new}}\right)^2 \times \frac{MVA_b^{new}}{MVA_b^{old}}$$

Problem

1) A 300 MVA, 20 kV, 3$\phi$ generator has a subtransient reactance of 20%. The generator supplies 2 synchronous motors through a 64 km transmission line having transformers at both ends in [illegible] fig. $T_1$ is a 3$\phi$ transformer and $T_2$ is made of 3 single phase transformer of rating 100 MVA, 127/13.2 kV, 10% reactance. Series reactance of the transmission line is 0.5 $\Omega$/km. Draw the reactance diagram with all the reactances marked in p.u. Select the generator rating as base values.




Solution

$$MVA_b^{(new)} = 300 \text{ MVA}$$

$$kV_b^{(new)} = 20 \text{ kV}$$

Reactance of generator G : Since the generator rating and the base value are same, the generator p.u reactance does not change.

$\therefore$ p.u reactance of generator $= 20\% = 0.2$ p.u

ie;
$$X_{pu,G}^{(new)} = X_{pu}^{(old)} \times \left(\frac{kV_b^{old}}{kV_b^{new}}\right)^2 \times \frac{MVA_b^{new}}{MVA_b^{old}}$$

$$= 0.2 \times \left(\frac{20}{20}\right)^2 \times \frac{300}{300} = 0.2 \text{ pu}$$

Reactance of transformer $T_1$

$$X_{pu,T_1}^{(new)} = 0.1 \times \left(\frac{20}{20}\right)^2 \times \frac{300}{350} = 0.0857 \text{ p.u}$$

$0.1 \times \left(\frac{230}{\cdot}\right.$

Reactance of transmission line

Reactance of transmission line = 0.5 Ω / km

Total reactance of tr. line = 0.5 × 64 = 32 Ω

$$kV_b \text{ on HT side of } T_1 = kV_b \text{ on LT side} \times \frac{\text{HT voltage rating}}{\text{LT voltage rating}}$$

$$= 20 \times \frac{230}{20} = \underline{\underline{230 \text{ kV}}}$$

$$\text{Base impedance}, \; Z_b = \frac{(kV_b)^2}{MVA_b}$$

$$= \frac{230^2}{300} = 176.33 \; \Omega$$

$$\text{p.u reactance of Tr. line} = \frac{\text{Actual reactance}}{\text{Base impedance}}$$

$$= \frac{32}{176.33} = \underline{\underline{0.1815 \text{ p.u}}}$$

Reactance of Transformer $T_2$ :- The transformer $T_2$ is a 3-phase transformer bank formed using three numbers of single phase banks with voltage rating 127/13.2 kV. In this the high voltage side is star connected and low-voltage side is delta connected.

∴ voltage ratio of line voltage of 3-phase transformer bank

$$= \frac{\sqrt{3} \times 127}{13.2} = \frac{220}{13.2} \text{ kV}$$

$$kV_b \text{ on LT side of } T_2 = kV_b \text{ on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$= 230 \times \frac{13.2}{220} = \underline{\underline{13.8 \text{ kV}}}$$

The new p.u reactance of $T_2$

$$= 0.1 \times \left(\frac{13.2}{13.8}\right)^2 \times \frac{300}{(3 \times 100)} = \underline{\underline{0.0915 \text{ p.u}}}$$

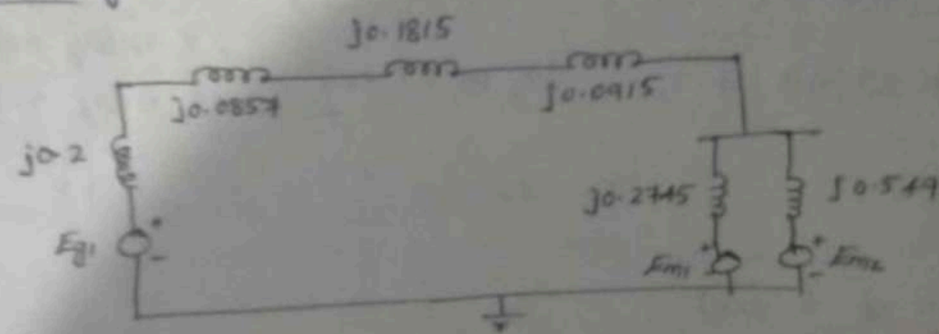
Reactance of $M_1$

$$X_{pu,M_1}^{new} = 0.2 \times \left(\frac{13.2}{13.8}\right)^2 \times \frac{300}{200} = 0.2745 \text{ p.u}$$

Reactance of $M_2$

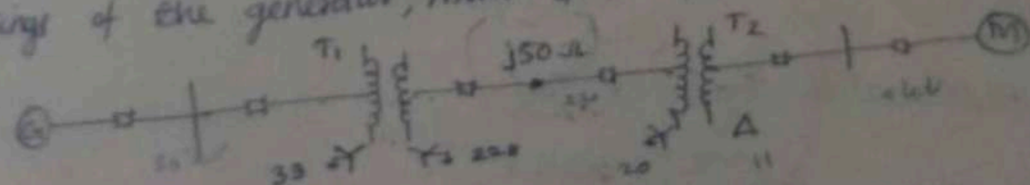
$$X_{p.u,M_2}^{new} = 0.2 \times \left(\frac{13.2}{13.8}\right)^2 \times \frac{300}{100} = 0.549 \text{ p.u}$$

Reactance diagram




Problem

2) Draw the reactance diagram for the power system shown in fig. Neglect resistance and use a base of 100 MVA, 220 kV in 50Ω line. The ratings of the generator, motor & transformers are given




Generator : 40 MVA , 25 kV , $x'' = 20\%$

Synchronous motor : 50 MVA , 11 kV , $x'' = 30\%$

Y-Y transformer : 40 MVA, 33/220 kV , $x = 15\%$

Y-Δ transformer : 30 MVA, 11/220 kV (Δ/Y) , $x = 15\%$

Solution

$$MVA_b^{new} = 100 \text{ MVA} \quad (\text{Same})$$

$$kV_b^{new} = 220 \text{ kV} \quad \text{in tr. line}$$

Other base kV :

(i) Generator side

$$kV_b \text{ on LT side of } T_1 = kV_b \text{ on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$= 220 \times \frac{33}{220} = 33\ kV$$

(ii) Motor side

$$kV_b \text{ on LT side of } T_2 = kV_b \text{ on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$= 220 \times \frac{11}{220} = 11\ kV$$

Reactance of tr. line

$$\text{Base impedance} = \frac{(kV_b^{new})^2}{MVA_b^{new}} = \frac{220^2}{100} = 484\ \Omega$$

$$\text{p.u reactance of tr. line} = \frac{\text{Actual reactance, } \Omega}{\text{Base impedance, } \Omega} = \frac{50}{484} = 0.1033\ p.u$$

Reactance of transformer $T_1$

$$kV_b^{new} = 33\ kV$$

$$\text{new p.u reactance of transformer } T_1 = 0.15 \times \left(\frac{33}{33}\right)^2 \times \frac{100}{40} = 0.375\ p.u$$

Reactance of generator G

$$X_{pu,G}^{new} = 0.2 \times \left(\frac{25}{33}\right)^2 \times \frac{100}{40} = 0.287\ p.u$$

Reactance of transformer $T_2$

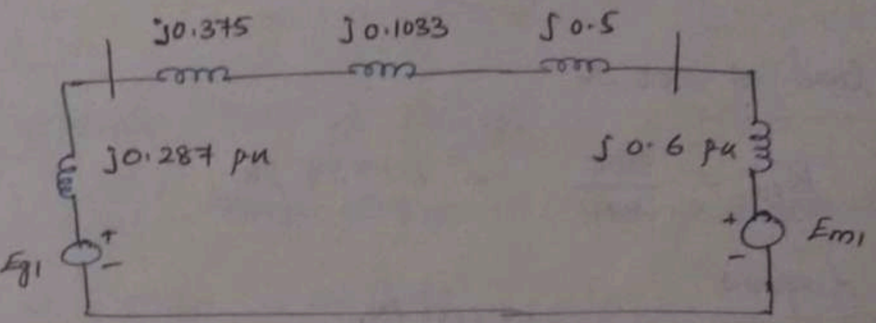
$$kV_b^{new} = 11\ kV$$

$$X_{p.u}^{new} = 0.15 \times \left(\frac{11}{11}\right)^2 \times \frac{100}{30} = 0.5\ p.u$$

Reactance of synchronous motor

$$X_{p.u}^{new} = 0.3 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{100}{50}\right) = 0.6\ p.u$$

Reactance diagram




All reactance values are in p.u

1) A system of unbalanced $3\phi$ voltages are given by $100V$, $j200V$ and $(-100-j160)V$. Determine the three symmetrical components of the system.

Sol

$$V_R = (100+j0)\ V = V_a$$

$$V_Y = (0+j200)\ V = V_b$$

$$V_B = (-100-j160)\ V = V_c$$

and phase sequence is RYB

→ The zero sequence components of 'R' phase is given by,

$$V_0 = \frac{1}{3}(V_R + V_Y + V_B) = \frac{1}{3}(V_a+V_b+V_c)$$

$$= \frac{1}{3}(100 + j200 - 100 - j160) = j13.33\ V$$

$$V_1 = \frac{1}{3}(V_R + aV_Y + a^2 V_B) = \frac{1}{3}(V_a + aV_b + a^2 V_c)$$

$$= \frac{1}{3}\left[100 + (-0.5+j0.866)(j200) + (-0.5-j0.866)(-100-j160)\right]$$

$$= \frac{1}{3}\left[100 - j100 - 173.2 + 50 + j80 + j86.6 - 138.56\right]$$

$$= \frac{1}{3}(-161.76 + j66.6) = (-53.92 + j22.2)\ V$$

$a = 1\angle 120^\circ = e^{j120} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = -0.5 + j0.866$

$$V_2 = \frac{1}{3}(V_R + a^2 V_Y + aV_B) = \frac{1}{3}(V_a + a^2 V_b + aV_c)$$

$$= \frac{1}{3}\left[100 + (-0.5-j0.866)(j200) + (-0.5+j0.866)(-100-j160)\right]$$

$$= \frac{1}{3}(461.76 - j106.6) = (153.92 - j35.53)\ V$$

→ The sequence component of all phases are given by

$$V_0 = (0 + j13.33)\ V$$

$$V_1 = (53.92 + j22.2)\ V$$

$$V_2 = (153.92 - j35.53)\ V$$

2) The phase 'A' of a $3\phi$ system feeding a Delta connected load gets open circuited. currents in B and C phases are 1 and -1. Determine the sequence components of the currents.

Sol

Positive sequence current $= \frac{1}{3}(I_a + aI_b + a^2 I_c)$

$= \frac{1}{3}[0 + a(+1) + a^2(-1)] = \frac{1}{3}(a - a^2)$

$= \frac{1}{3}(-0.5 + j0.866 + 0.5 + j0.866) = j\frac{1}{\sqrt{3}}$

Negative sequence currents

$= \frac{1}{3}(I_a + a^2 I_b + a I_c)$

$= \frac{1}{3}\ 0 + a^2(1) - a(-1) = \frac{-j}{\sqrt{3}}$

Zero sequence current is zero, because

$I_a + I_b + I_c = 0$

3) The 3 currents in an unbalanced $3\phi$ system are $I_a = 0 + j10$, $I_b = 10 + j0$, $I_c = 0$. Find the sequence currents and check the result by inverse transform.

Sol

positive sequence current

$I_1 = \frac{1}{3}(I_a + aI_b + a^2 I_c)$

$= j10 + (-5) + j8.66 + 0] / 3 = -1.67 + 6.22j$

Negative sequence current

$I_2 = \frac{1}{3}(I_a + a^2 I_b + a I_c)$

$= [j10 - 5 - j0.866 + 0] / 3 = -1.67 + 0.447j$

Zero sequence current

$$I_0 = \frac{1}{3}(I_a + I_b + I_c) = 3.33 + j3.33$$

→ As a check, we calculate $I_a, I_b, I_c$ from the sequence currents by substituting in equations.

$$I_a = I_1 + I_2 + I_0$$

$$I_b =$$

$$I_a = I_0 + I_1 + I_2$$

$$I_a = (-1.67 + 6.22j) + (-1.67 + j0.447) + (3.33 + j3.33)$$

$$= 0 + 10j$$

$$I_b = I_0 + a^2 I_1 + a I_2$$

$$I_b = (-1.67 + j6.22)\,a^2 + (-1.67 + j0.447) + (3.33 + j3.33)$$

$$= (-1.67 + 6.22)\,j\,(-0.5 - j0.866) + (-1.67 + j0.447)(-0.5 + j0.866) + (3.33 + j3.33)$$

$$= 10.02 + j0$$

$$= I_0 + a I_1 + a^2 I_2$$

$$I_c = (-1.67 + 6.22j)\,a + (-1.67 + 0.447j)\,a^2 + (3.33 + j3.33)$$

$$= (-1.67 + 6.22j)(0.5 - j0.866) + (-1.67 + 0.447j)(-0.5 - j0.866) + (3.33 + j3.33)$$

$$= 0 + j0$$

→ Thus the result is checked.

3) The voltages across a 3 phase unbalanced load are $V_a = 200\angle 40^\circ$, $V_b = 320\angle 190^\circ$, $V_c = 480\angle 340^\circ$. Determine the symmetrical components of voltages. phase sequence is abc.

→ The +ve sequence components are

$V_1 = V_1$ $\quad V_1 = a^2 V_1$ $\quad V_1 = a V_1$

$V_1 = 141.735 \angle -91.186^\circ = -2.933 - j\,141.7$

$V_1 = 141.735 \angle 148.814^\circ = -121.253 + j\,73.39$

$V_1 = 141.7353 \angle 28.814 = 124.18 + j\,68.312$

→ The -ve sequence components are

$V_2 = V_2$ $\quad V_2 = a V_2$ $\quad V_2 = a^2 V_2$

$V_2 = 306.538 \angle 78.75^\circ = 59.768 + j\,300.655$

$V_2 = 306.538 \angle 198.75^\circ = -290.26 - j\,98.567$

$V_2 = 306.538 \angle 318.756 = 230.49 - j\,202.088.$

A) The symmetrical components of phase a voltage in a 3 phase unbalanced system are $V_0 = 25\angle 0^\circ$, $V_1 = 60\angle 90^\circ$ and $V_2 = 30\angle 180^\circ$. Determine the phase voltages $V_a$, $V_b$ and $V_c$

Sol

The phase voltages of $V_a, V_b, V_c$ are given by following matrix eqn

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}$$

$V_a = V_0 + V_1 + V_2$

$V_b = V_0 + a^2 V_1 + a V_2$

$V_c = V_0 + a V_1 + a^2 V_2$

$V_a = 60\angle 90^\circ + 25\angle 0^\circ + 30\angle 180^\circ$

$= -5 + j60 \quad = 60.208\angle 94.763^\circ$

$V_b = 25\angle 0^\circ + (1\angle 240^\circ \times 60\angle 90^\circ) + 1\angle 120^\circ \times 30\angle 180^\circ$

$= 25 + j0 + 51.961 - j30 + 15 - j25.980$

$= 91.961 - j55.980 \quad = 107.66\angle -31.33^\circ$

$V_c = [25\angle 0^\circ + (1\angle 120^\circ \times 60\angle 90^\circ) + (1\angle 240^\circ \times 30\angle 180^\circ)$

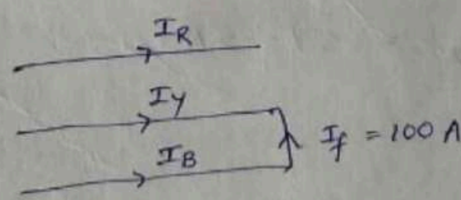
$= 25 + j0 - 51.96152 - j30 + 15 - j25.98$

$= -11.961 - j4.019$

5) Determine the magnitude of the symmetrical components $(I_0, I_1, I_2)$ of the currents in a $3\phi$ 3 wire systems, when a short circuits occurs between R and Y phase wire, the fault current being 100 ampere.

soln

Consider the sequence of phase is RYB as shown.




Here boundary conditions are

$I_R = 0 \quad , \quad I_Y + I_B = 0 \quad , \quad I_Y = -I_B = I_f = 100A$

→ The zero sequence component of the current is given by

$I_0 = \frac{1}{3}(I_R + I_Y + I_B)$

$= \frac{1}{3}[0 + (-I_B) + I_B] \quad = 0$

$\because I_Y = -I_B$

→ The +ve sequence component of the current is given by.

$$I_1 = \frac{1}{3}(I_R + a I_Y + a^2 I_B)$$

$$= \frac{1}{3}(0 + a I_Y - a^2 I_Y)$$

$$= \frac{1}{3}(0) + \frac{1}{3} I_Y (a - a^2)$$

$$= \frac{1}{3}(-0.5 + j0.866 + 0.5 + j0.866) \times 100$$

$$I_1 = j57.73 \text{ A} \qquad = 57.73\angle 90^\circ$$

$$|I_1| = 57.73 \text{ A}$$

→ Now the -ve seq. components of the currents is given by

$$I_2 = \frac{1}{3}(I_R + a^2 I_Y + a I_B)$$

$$= \frac{1}{3}(0 + a^2 I_Y - a I_Y)$$

$$= \frac{1}{3}(a^2 - a) I_Y \Rightarrow \frac{1}{3}(-0.5 - j0.866 + 0.5 - j0.866) \times 100$$

$$= -j57.73 \text{ A} \qquad = 57.73\angle -90^\circ$$

$$|I_2| = 57.73 \text{ A}$$

Reference: Modern Power s/m Analysis : Nagrath
Power s/m Analysis : Nagoorkani

# Short Circuit Analysis

Fault is any failure which interfers with normal flow of current. Fault can be due to

* Insulation failure
* Flash over by lightning.
* Permanent damage to conductors and towers.
* Accidental faulty operation.

## Broad classification of fault.

* Shunt fault – short circuit fault → Symmetrical / Unsymmetrical
* Series fault – Open circuit fault → one open conductor fault / two open conductor fault.

Shunt fault or short circuit fault is associated with increase in current and decrease in voltage and frequency.

Series fault or open circuit fault is associated with increase in voltage and frequency and decrease in current.

* Symmetrical fault
* Unsymmetrical fault.

Symmetrical fault is associated with change in volt or current in all the three phases. Also called $3\phi$ fault.

Unsymmetrical fault is associated with change in volt or current different in all the three phases. Unsymmetrical faults can be

- Single line to ground fault (SLG) or Line to ground
- Line to Line fault (L-L)
- Double line to ground fault ( ~~Dt~~ LLG)

The symmetrical fault conditions are analyzed on per phase basis using thevenins theorem or using bus impedance matrix. Unsymmetrical faults are analyzed using symmetrical components.

70 to 80 % of faults are SLG faults, 5% are $3\phi$ faults. 10% L-L-G faults, 15% L-L faults.

Fault calculations.

When fault occur in a part of power system, heavy current flows in the part of circuit, which may cause permanent damage to the equipments. Hence faulty part should be isolated from healthy part immediately on the ocurance of the fault. This can be achieved by using protective relays and circuit breakers. Relays sense faulty conditions and send signals to circuit breakers to open the circuit under faulty conditions.

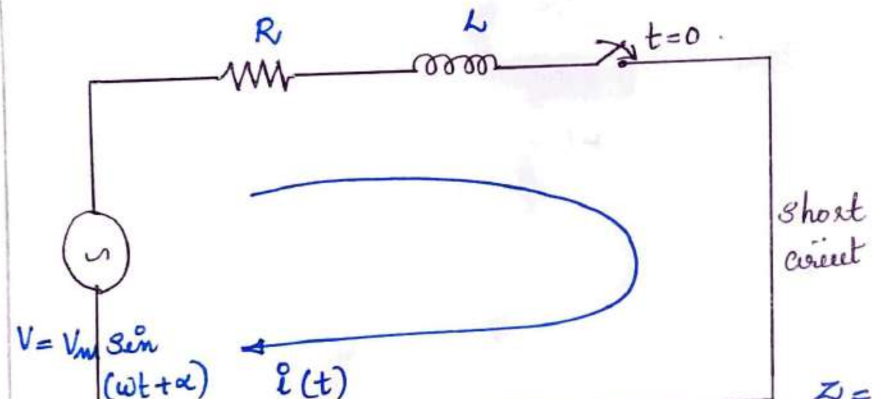
The current flowing in different parts of power s/m immediately after the fault differs from the current flowing after few cycles, from the ocurance of fault. The selection of CB, is based on the immediate current. The estimation of these currents for various types of faults at various location of the system are ~~fs~~ called fault calculations.

## ...nsients due to Short Circuit in transmission line.

Most of the components of the power system have inductive property which give rise to transients where there is a sudden change in current. Faults on power system are accompanied by sudden change in current which give rise to transients condition in power system.

Assumptions :

* Line is fed from constant voltage source.
* Short circuit takes place when it is not loaded.
* Line capacitance is negligible and line is represented by series resistance and inductance.




Let ;

$i(t)$ = Current in transmⁿ line under short ckt condⁿ.

$Z$ = Impedance of transⁿ line

$$Z = R + j\omega L = \sqrt{R^2+\omega^2 L^2} \angle \tan^{-1} \omega L/R$$
$$= |Z| \angle \theta$$

where $|Z| = \sqrt{R^2+\omega^2 L^2}$ ; $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$.

Let fault occurs when $t=0$. When switch is closed at $t=0$ fault current (short circuit current) flows in the circuit. The KVL eqⁿ for the loop is

$$R\, i(t) + L \frac{di(t)}{dt} = V_m \sin(\omega t + \alpha)$$

The solution of the above equation gives,

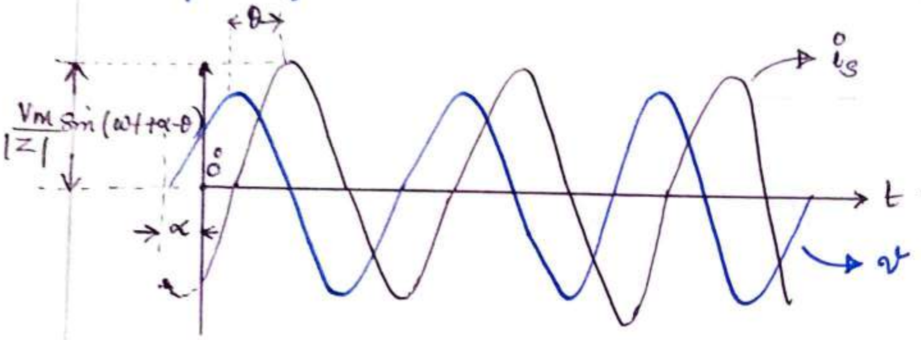
$$i(t) = \underbrace{\frac{V_m}{|Z|}\sin(\omega t + \alpha - \theta)} + \underbrace{\frac{V_m}{|Z|}\sin(\theta - \alpha)\, e^{-R/L\, t}}$$

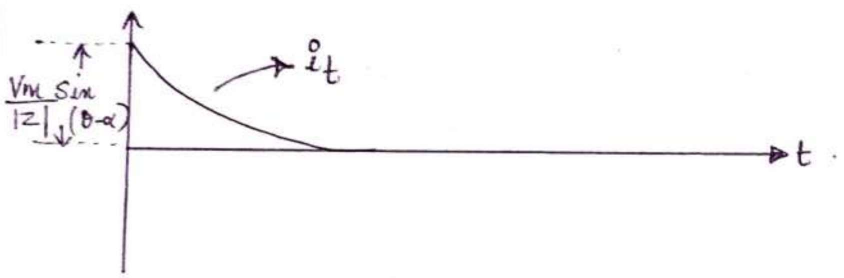
Symmetrical short circuit current or steady state current $i_s$.

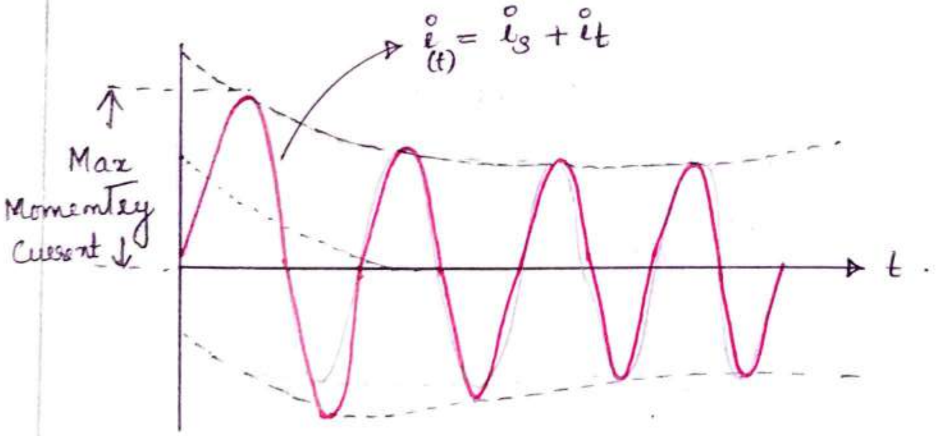
DC offset current or transient current ($i_t$)

$$\therefore \quad i(t) = i_s + i_t$$

The plot for $i(t)$, $i_s$, $i_t$ and $v$ with respect to $t$ are shown.




From the equation, it can be seen that the short circuit component of current has two components, a steady state sinusoidal component and unidirectional transient component.

(3)

S the short circuit component the value corresponding to the first peak is called max momentary short circuit current, $i_{mm}$. The first peak obtained when $\sin(\omega t + \alpha - \theta) = 1$.

$$\therefore \quad i_{mm} = \frac{V_m}{|Z|} + \frac{V_m}{|Z|} \sin(\theta - \alpha) e^{-R/L \cdot t}$$

if the time t is very less, take $t \simeq 0$

$$\therefore \quad i_{mm} = \frac{V_m}{|Z|} + \frac{V_m}{|Z|} \sin(\theta - \alpha)$$

In transmission line, usually resistance very less compared to inductance, hence $\theta \simeq 90^\circ$.

$$\therefore \quad i_{mm} = \frac{V_m}{|Z|} + \frac{V_m}{|Z|} \sin(90 - \alpha).$$

$$= \frac{V_m}{|Z|} + \frac{V_m}{|Z|} \cos\alpha$$

$i_{mm}$ has max possible value when $\alpha = 0^\circ$.

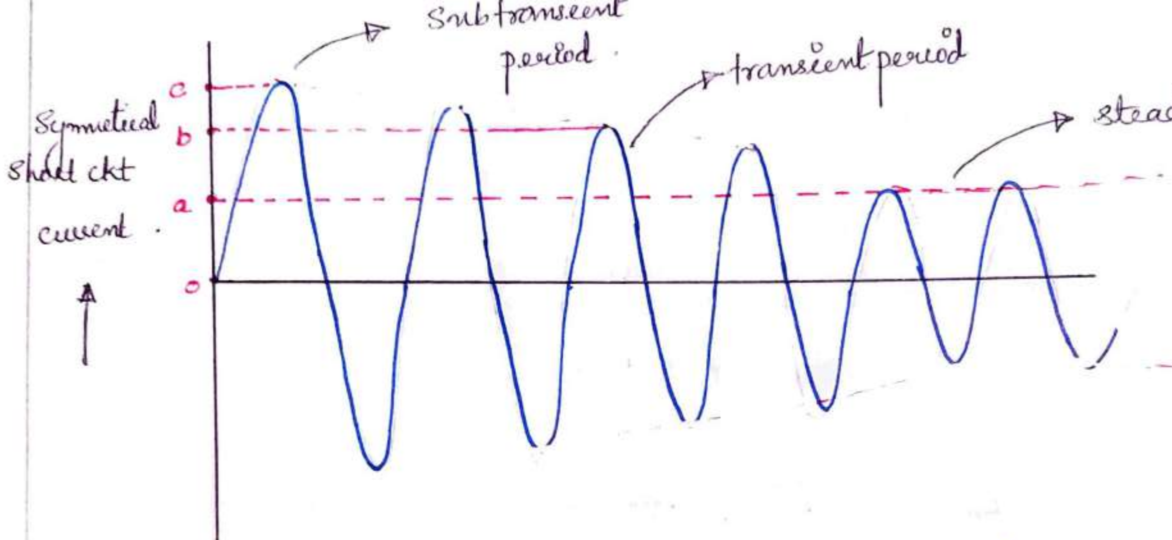
$$\therefore i_{mm} = \frac{2V_m}{|Z|}.$$

$\therefore$ The maximum value of short circuit current is double the value of symmetrical short ckt current. If such a condition exists in a transmission line, that effect is called doubling effect. A safer choice of momentary current rating of C.B can be obtained by maximum possible value of short circuit current.

## Transients due to short circuit in 3ϕ Alternator (Synchronous ...

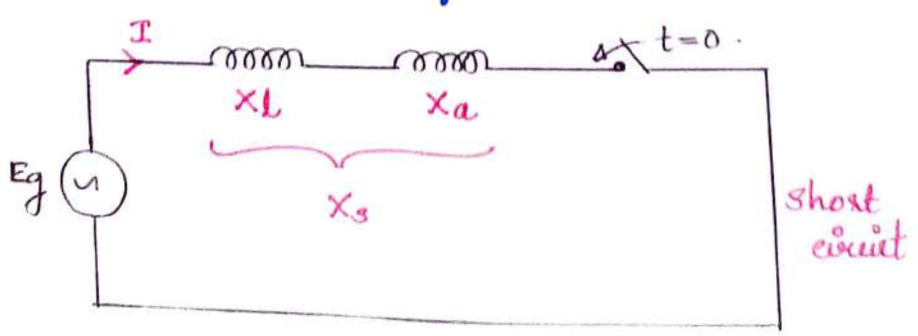
unloaded of 'n' rel...

Consider a three phase alternator running at no load is called ... If a 3ϕ fault occurs at the terminals of the alternator, then a heavy component short circuit current flows in the armature circuit. The oscillogram of set the short circuit current after removing the DC offset current is shown.




The symmetrical short circuit current can be divided into three regions, called subtransient, transient and steady state region.

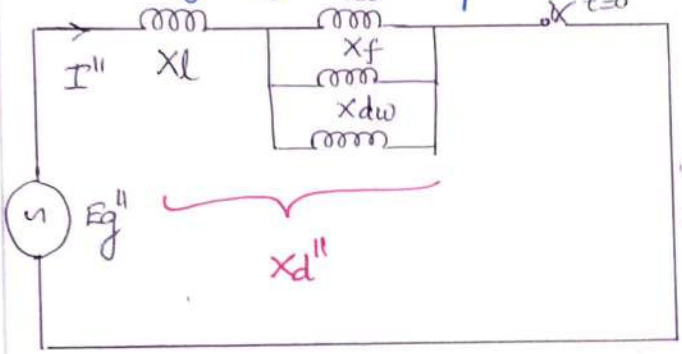
Under steady state short circuit condition, the armature reaction of a synchronous generator produces a demagnetising flux. This effect is represented as reactance called armature reactance reac... $X_a$. The sum of leakage and armature reaction reactance is called synchronous reactance. ($X_s$). On neglecting the armature resistance, the steady state short circuit model of an alternator is as shown.



During a fault occurs, a sudden increase in current occurs, which appears in all the three phases of the alternator. This increases field current and damper winding current. This effect can be represented by two reactances in parallel with $X_a$ as shown. $X_f$ represents reactance due to field winding and $X_{dw}$ represents the reactance of damper winding. The total reactance under this condition

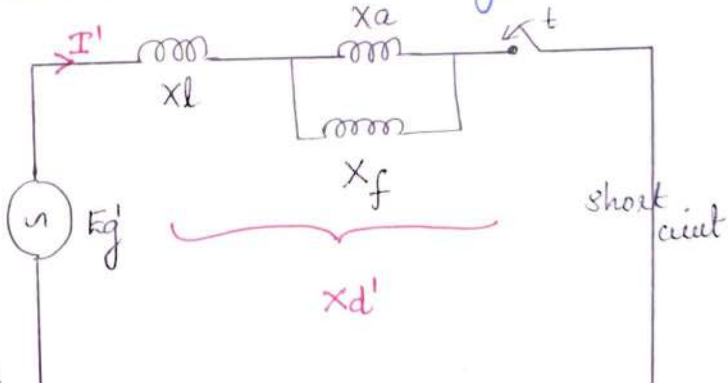
$$X_d'' = \frac{1}{\frac{1}{X_a} + \frac{1}{X_{dw}} + \frac{1}{X_f}} + X_l$$

The reactance is less and hence the short circuit current during this period is very high. This period is called subtransient period.




The induced current in the damper windings disappear after few cycles from the instant of fault, due to the small time constant of damper winding than field winding. This effect is equivalent to open circuit the damper winding reactance. This state is called transient state; and is denoted by $X_d'$.

$$\therefore \quad X_d' = X_l + \frac{1}{\frac{1}{X_a} + \frac{1}{X_f}}$$




The transients state will last for few cycles, and then steady state conditions are reached. Effect of field winding also die out in short time depending on time constant.

$$\therefore \quad X_d = X_a + X_l$$

From the reactanus obtained, $X_d'' < X_d' < X_d$.

Let $I$ = Rms value of steady state current

$I'$ = Rms value of transient current

$I''$ = Rms value of subtransient current.

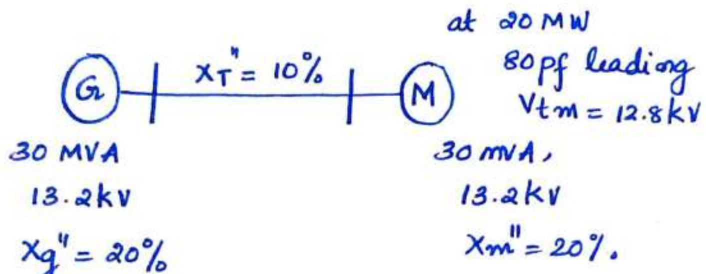
$$I = \frac{oa}{\sqrt{2}} \; ; \; I' = \frac{ob}{\sqrt{2}} \; ; \; I'' = \frac{oc}{\sqrt{2}}$$

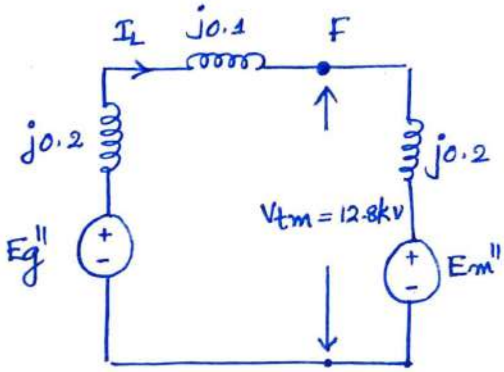
$$\therefore \; X_d'' = \frac{E_g}{I''} \; ; \; X_d' = \frac{E_g}{I'} \quad X_d = \frac{E_g}{I}$$

The momentey current rating of the circuit breakers used for generats and motors are determined using subtransient reactanees.

Synchronous generator and motor are rated for 30,000 kVA, 13.2 kV and both have subtransient reactance of 20%. The line connecting them has a reactance of 10% on the base of machine ratings. The motor is drawing 20,000 kW at 0.8 pf leading. The terminal voltage of the motor is 12.8 kV. When a symmetrical 3$\phi$ fault occurs at motor terminals, find the subtransient current in generator, motor and at the fault point.

sol$^n$:




Base MVA = 30

kV = 13.2 kV

Terminal voltage at motor terminal = 12.8 kV

$$\therefore \text{ p.u value of } \text{"} = \frac{12.8}{13.2} = \underline{\underline{0.969 \text{ p.u}}}$$

$I_L$ = load current = Current drawn by motor.

P of motor = 20 MW and p.f = 0.8, voltage at terminal $V_{tm} = 12.8$ kV

$$\therefore \quad I_L = \frac{20,000}{12.8 \times 0.8 \times \sqrt{3}} = 1127.6 \text{ A} \quad (1127.6)$$

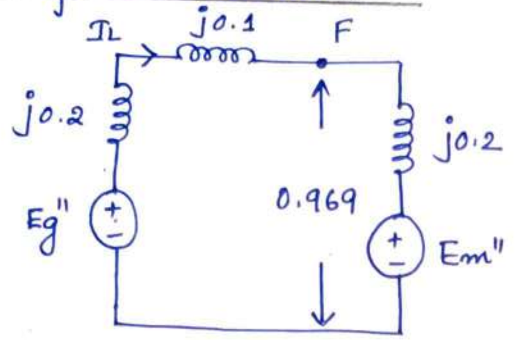
$$\text{Base Current} = \frac{(\text{MVAb}) \times 1000}{\sqrt{3} \times \text{kVb}} = \frac{30 \times 1000}{\sqrt{3} \times 13.2} = 1312.16 \text{ A}.$$

$$\therefore \text{ p.u value of load current, } I_L = 1127.6 / 1312.16 = \underline{\underline{0.859 \text{ p.u}}}$$

$$\text{Angle of } I_L = \cos^{-1}(0.8) = 36.86^\circ$$

$$\therefore \quad I_L = 0.859 \angle 36.86^\circ \text{ p.u} \quad ; \quad V_{tm} = 0.969 \angle 0^\circ \text{ p.u.}$$

Prefault Condition :




Prefault voltage at fault point

$$= V_{tm} = 0.969 \text{ p.u}$$

$$I_L = 0.859 \angle 36.86^\circ \text{ p.u}$$

$$E_g'' = V_{tm} + I_L \cdot (j0.2) + I_L(j0.1)$$

$$= 0.969 + 0.859 \angle 36.86^\circ (j0.3) = 0.8144 + j0.2061$$

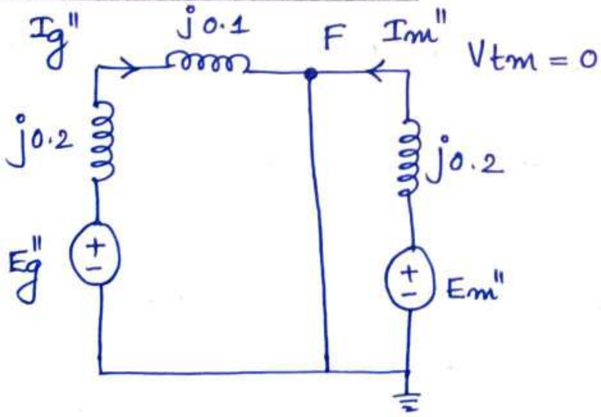
$$= 0.840 \angle 14.2^\circ \text{ p.u}$$

$$E_m'' = V_{tm} - j0.2 \cdot I_L$$

$$= 0.969 - 0.859 \angle 36.86^\circ (j0.2) = 1.072 - 0.137j$$

$$= 1.08 \angle -7.3^\circ \text{ p.u}$$

Fault Condition :-




$$I_g'' = \frac{E_g''}{j0.2 + j0.1} = \frac{0.840 \angle 14.2^\circ}{j0.3}$$

$$= 2.8 \angle -75.8^\circ \text{ p.u}$$

$$I_m'' = \frac{E_m''}{j0.2} = \frac{1.08 \angle -7.3^\circ}{j0.2}$$

$$= 5.4 \angle -97.3^\circ$$

...ult Current $I_f'' = I_g'' + I_m''$

$$= 2.8\angle -75.8^\circ + 5.4\angle -97.3^\circ$$

$$= \underline{\underline{8.07\angle -89.9^\circ}}\ p.u$$

$\therefore$ To find Actual values of fault current, multiply the p.u values with base values.

$\therefore$ Fault Current $= 8.07\angle -89.9^\circ \times 1312.16$

$$= \underline{\underline{10.58\angle -89.9^\circ}}\ kA$$

Subtransient current in Generator $= 2.8\angle -75.8^\circ \times 1312.16$

$$= \underline{\underline{3.674\angle -75.8^\circ}}\ kA$$

Subtransient Current in Motor $= 5.4\angle -97.3^\circ \times 1312.16$

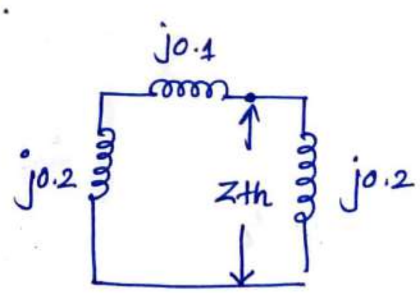
$$= \underline{\underline{7.085\angle -97.3^\circ}}\ kA$$

Using Thevenins theorem.

To use Thevenins theorem, we need prefault calculation which we have already done. The voltage (prefault) at the fault point is the thevenins voltage.

$\therefore V_{th} = V_{tm} = 0.969\angle 0^\circ$.

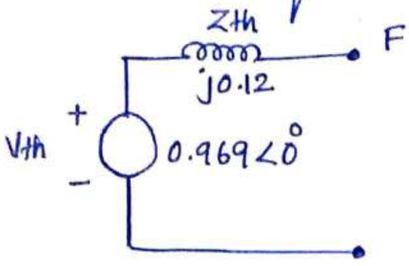
To find Zth; look from the point of fault.

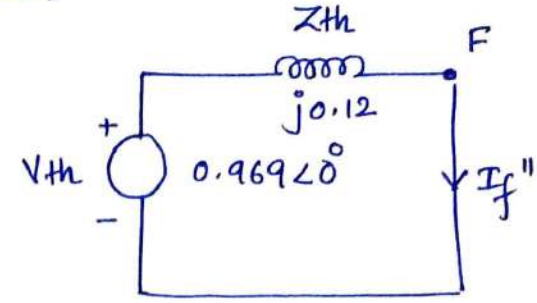



$\Rightarrow Z_{th} = (j0.2 + j0.1) \,//\, j0.2$

$= \underline{\underline{j0.12}}$

$\therefore$ Thevenins equivalent circuit is ;



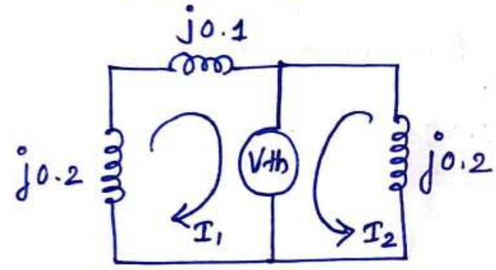




Prefault

$$\therefore \quad I_f'' = \frac{V_{th}}{Z_{th}} = \frac{0.969\angle 0^\circ}{j0.12} = \underline{\underline{8.075\angle -90^\circ}} \text{ p.u}$$

Before fault current in Generator & motor $= I_L = 0.859\angle 36.86^\circ$

After fault, the change in current in Generator and motor can be calculated as:




$\therefore \quad I_1$ = change in current in generator

$$= \frac{V_{th}}{j0.2 + j0.1} = \frac{0.969}{j0.3}$$

$$= \underline{\underline{3.23\angle -90^\circ}} \text{ p.u}$$

$I_2$ = change in current in motor

$$= \frac{V_{th}}{j0.2} = \frac{0.969}{j0.2} = 4.84\angle -90^\circ \text{ p.u}$$

$\therefore$ Current in Generator $=$ prefault volt $+ I_1$

$$= 0.859\angle 36.86^\circ + 3.23\angle -90^\circ.$$

$$= \underline{2.86\angle -76.1^\circ} \text{ p.u}$$

$\therefore$ Current in Motor $= I_2 -$ prefault volt

$$= 4.84\angle -90^\circ - 0.859\angle 36.86 = \underline{5.39\angle -97.3^\circ}$$

* Calculate the actual values in the same way as done earlier

## Unsymmetrical Fault Analysis.

Various types of unsymmetrical faults that occur in power system are :-

shunt type faults :

(i) Single line to ground fault (LG) fault

(ii) Line to Line fault (L-L)

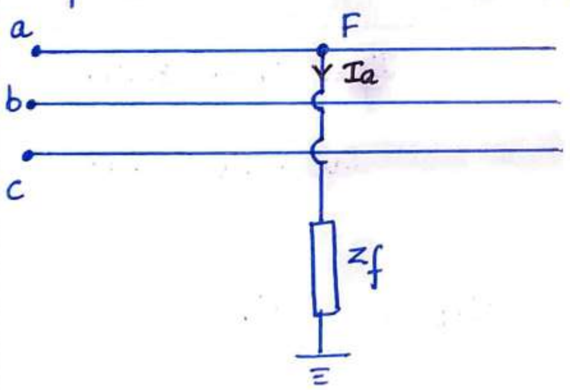
(iii) Double line to ground fault (L-L-G) fault.

Series type faults :

Open Conductor faults (one or two conductors open) fault.

## Single Line to ground fault (LG fault).

Consider the network shown, where a L-G fault occurs at phase 'a' at F in the s/m, through a fault impedance $Z_f$.




At the fault point F, the currents and voltages are considered as;

$I_b = 0$

$I_c = 0$

$V_a = I_a \cdot Z_f$.

$$\therefore \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

$\therefore \quad I_{a0} = \frac{1}{3} I_a \; ; \quad I_{a1} = \frac{1}{3} I_a \; ; \quad I_{a2} = \frac{1}{3} I_a$

$$\therefore \quad I_{a0} = I_{a1} = I_{a2} = \frac{1}{3} I_a \quad \longrightarrow (1)$$

$\therefore \; I_a = 3 I_{a1}$

$= 3 I_{a0}$

$= 3 I_{a2}$.

we have

$V_a = Z_f . I_a$

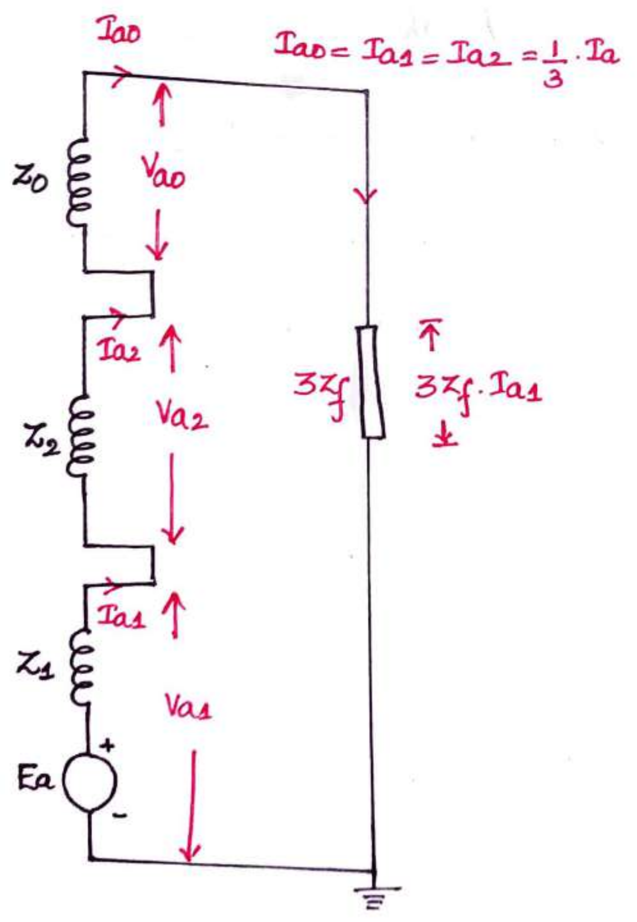
$V_a = V_{a0} + V_{a1} + V_{a2}$ (in terms of symmetrical components).

$$\therefore \quad V_{a0} + V_{a1} + V_{a2} = Z_f . I_a$$

$$= Z_f . 3 I_{a1}$$

$$= 3 . Z_f . I_{a1} \quad \longrightarrow (2).$$

On Analysing eq$^n$ (1). ie; $I_{a0} = I_{a1} = I_{a2}$; zero, sequence, +ve and -ve sequence network are connected in series. On Including eq$^n$ (2), the resultant network representing L-G fault can be drawn as;




$\therefore$ In LG fault.

$$\boxed{\begin{array}{c} I_f = I_a = 3 . I_{a1} \\ I_{a1} = I_{a2} = I_{a0} \end{array}}$$

$$\boxed{I_{a1} = \frac{E_a}{Z_1 + Z_2 + Z_0 + 3 Z_f}}$$

$I_f = 3 . I_{a1}$

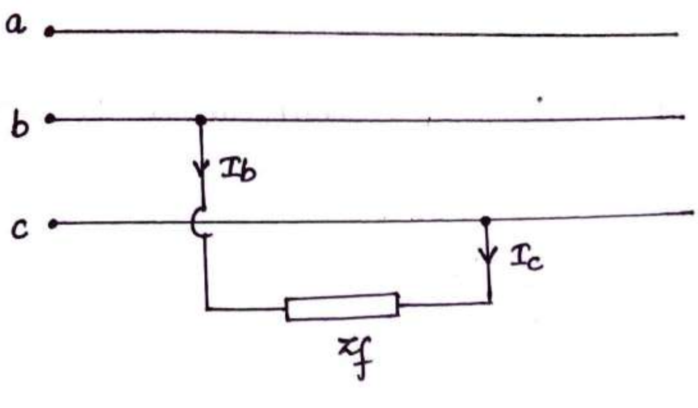
$V_{a1} = E_a - I_{a1} Z_1$

$V_{a2} = - I_{a2} Z_2$

$V_{a0} = - I_{a0} . Z_0$

## to Line fault (L-L fault).

Figure shows a line to line fault in a power systems on phases b and c through a fault Impedance $Z_f$.




$\therefore$ here $I_a = 0$

$I_b = I_b$

$I_c = -I_b$

$V_b - V_c = I_b \cdot Z_f$.

$\therefore$ Symmetrical Components of current are;

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

$\therefore I_{a0} = 0$

$I_{a1} = \frac{1}{3}(\alpha I_b - \alpha^2 I_b)$

$I_{a2} = \frac{1}{3}(\alpha^2 I_b - \alpha I_b)$

$\therefore I_{a2} = -I_{a1}$. $\rightarrow$ (1)

Considering voltages.

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_b - Z_f \cdot I_b \end{bmatrix}$$

$$V_{a1} = \frac{1}{3}\left(V_a + \alpha V_b + \alpha^2 (V_b - Z_f \cdot I_b)\right)$$

$$V_{a2} = \frac{1}{3}\left(V_a + \alpha^2 V_b + \alpha (V_b - Z_f \cdot I_b)\right)$$

$$V_{a1} - V_{a2} = \left(\tfrac{1}{3}(\alpha - \alpha^2) V_b + (\alpha^2 - \alpha)(V_b - Z_f \cdot I_b)\right)$$

$$= \tfrac{1}{3}(1.73\angle 90^\circ V_b + 1.73\angle -90^\circ \cdot V_b - 1.73\angle -90^\circ Z_f \cdot I_b)$$

$$= -(\tfrac{1}{3} 1.73\angle -90^\circ Z_f \cdot I_b)$$

$$\left\{ \begin{aligned} I_b &= I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2} \\ &= 0 + (\alpha^2 - \alpha) \cdot I_{a1} \end{aligned} \right\}$$

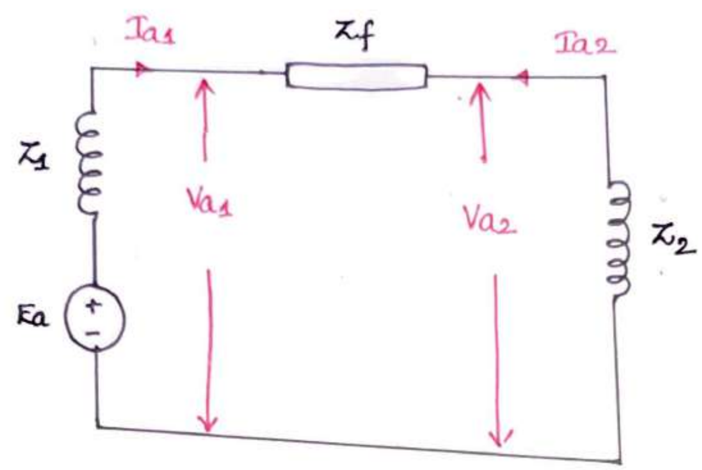
$$= 1.73\angle -90^\circ I_{a1}$$

$$V_{a1} - V_{a2} = Z_f \cdot I_{a1}. \rightarrow (2)$$

On analyzing eq$^n$ (1) and (2) i.e; $I_{a1} = -I_{a2}$ and I

$V_{a1} - V_{a2} = Z_f . I_{a1}$

the resultant circuit representing fault can be drawn as




fault Current $I_f = \cancel{I_{a1}}\ I_b$

Fault Current $I_f = I_b$

$$I_b = I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2}$$

here;

$$I_{a1} = -I_{a2}$$

$$I_{a0} = 0$$

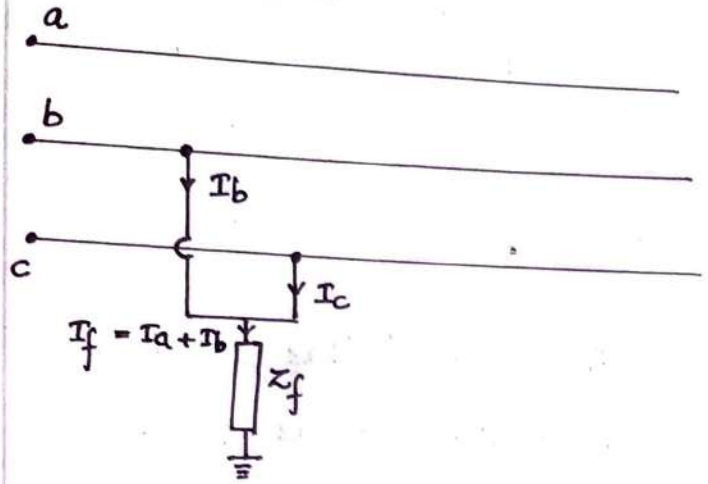
$$\therefore \boxed{I_f = (\alpha^2 - \alpha) . I_{a1} \qquad I_{a1} = \frac{E_a}{Z_1 + Z_2 + Z_f}}$$

$$V_{a1} = E_a - I_{a1} . Z_1 .$$

$$\boxed{V_{a1} - V_{a2} = I_{a1} . Z_f .}$$

## ...uble line to Ground Fault (L-L-G fault).

Figure shows a double line to ground fault. Let the fault occurs through a fault impedance $Z_f$. Let the fault occurs on phases b and c.




$\therefore$ here $I_a = 0$

$I_f = I_c + I_b$

$V_b = V_c = I_f \cdot Z_f$

Symmetrical Components of current can be written as

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} 0 \\ I_b \\ I_c \end{bmatrix} \qquad \therefore \quad I_{a0} + I_{a1} + I_{a2} = 0$$

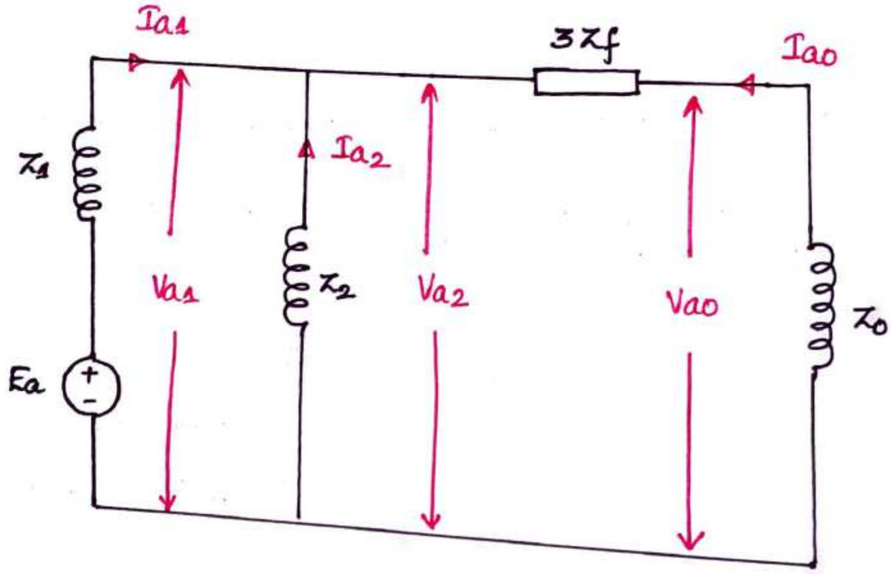
$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_b \end{bmatrix}$$

$$V_{a1} = \frac{1}{3}\left( V_a + \alpha V_b + \alpha^2 V_b \right)$$

$$V_{a2} = \frac{1}{3}\left( V_a + \alpha^2 V_b + \alpha V_b \right) \qquad V_{a0} = \frac{1}{3}\left( V_a + 2V_b \right)$$

$$\therefore \quad V_{a1} = V_{a2} \rightarrow (1) \qquad V_{a0} - V_{a1} = 3 \cdot Z_f \cdot I_{a0} \rightarrow (2)$$

On analyzing equations (1) and (2); the circuit representing L-L-G fault can be drawn as




$\therefore$ here $V_{a1} = V_{a2}$

$V_{a0} - V_{a1} = 3Z_f I_{a0}$

$I_{a1} + I_{a2} + I_{a0} = 0$

$\boxed{\text{Fault Current } I_f = I_b + I_c}$ ; $I_b = I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2}$

$I_c = I_{a0} + \alpha I_{a1} + \alpha^2 I_{a2}$

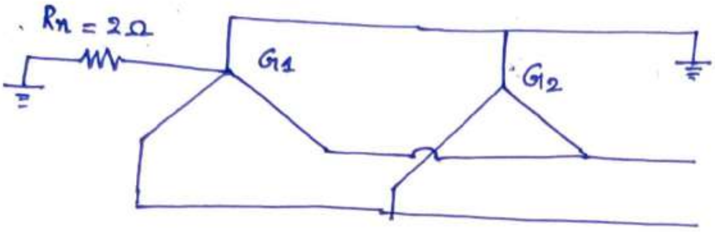
$$\boxed{I_{a1} = \frac{E_a}{Z_1 + \left( Z_2 \,||\, (3Z_f + Z_0) \right)}}$$

$$V_{a1} = E_a - I_{a1} \cdot Z_1$$

$$= V_{a2}$$

$$= - I_{a2} \cdot Z_2$$

Two 11 kV, 20 MVA, three phase, star connected generators operate in parallel as shown in figure. The +ve, −ve and zero sequence reactances of each being respectively j0.18, j0.15, j0.10 p.u. The star point of one of the generator is isolated and that of the other is earthed through 2 Ω resistor. An L-G fault occurs at the terminals of one of the generator. Estimate (i) fault current (ii) Current in grounded resistor (iii) voltage across grounding resistor.




sol: step 1: Draw Sequence networks.

Choose $MVA_b = 20$ and $kV_b = 11$ ; Base impedance $= \frac{(kV_b)^2}{MVA_b} = \frac{11^2}{20}$

$= \underline{6.05\Omega}$

p.u value of $R_n = \frac{2}{6.05} = \underline{0.33 \text{ p.u}}$

given;

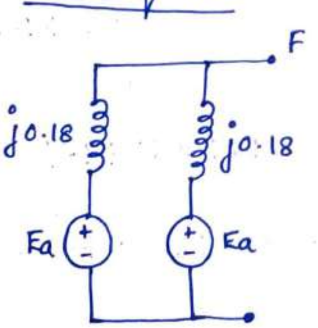
$X_{g1,1} = X_{g2,1} = j0.18$ p.u

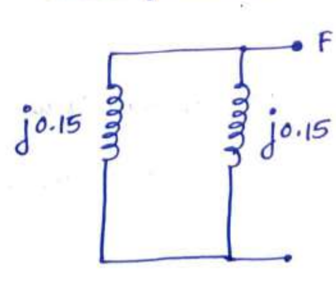
$X_{g1,2} = X_{g2,2} = j0.15$ p.u

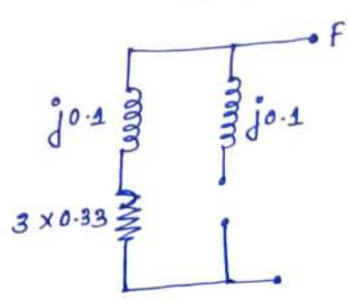
$X_{g1,0} = X_{g2,0} = j0.1$ p.u

$R_n = 0.33$ p.u

+ve Sequence: −ve Sequence Zero Sequence



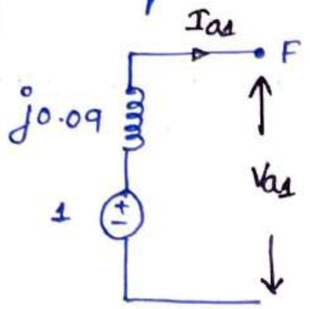



* The value of $E_a$ is taken as the pre-fault voltage $V_{pf}$

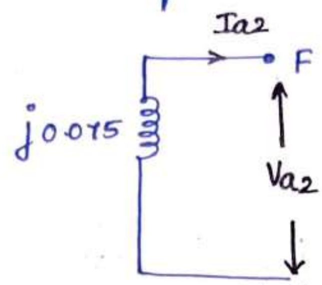
Take $V_{pf} = 1$ p.u.

$\therefore$ Equivalent Sequence n/w :

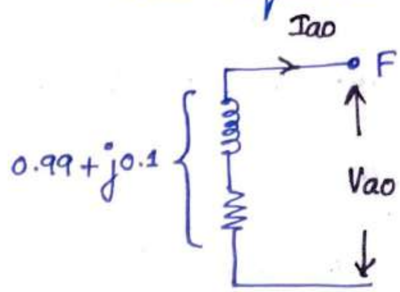
+ve Sequence



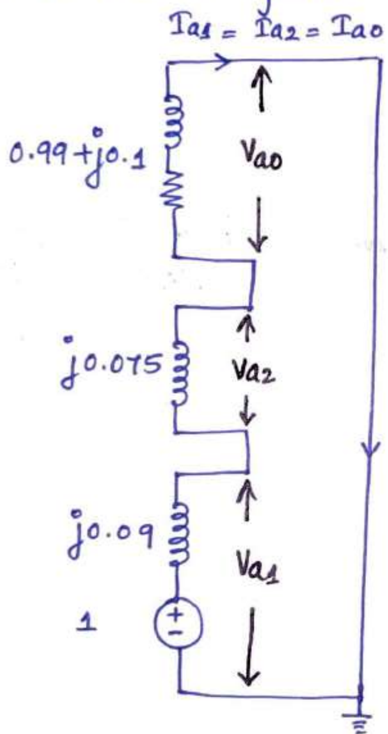

-ve Sequence




Zero Sequence




For a S.L.G fault :




here fault Current $I_f = I_a$

$I_a = 3 \cdot I_{a1}$

$$I_{a1} = \frac{V_{pf}}{j0.09 + j0.075 + 0.99 + j0.1}$$

$$= 0.975 \angle -14.9^\circ \text{ p.u}$$

(i)

$\therefore$ fault current $= 3 \times 0.975 \angle -14.9^\circ$

$$= \underline{\underline{2.927 \angle -14.9^\circ \text{ p.u}}}$$

Base Current $I_b = \dfrac{kVA_b}{\sqrt{3} \cdot kV_b} = \dfrac{20 \times 1000}{\sqrt{3} \times 11}$

$= 1049.7$ A

$\therefore$ Actual value of fault current $= 2.927 \times 1049.7$

$$= \underline{\underline{3.072 \angle -14.9^\circ \text{ kA}}}$$

Current through the neutral Resistor

is $I_{ao}$ ; for SLG $I_{no} = 3I_{ao}$

The current flowing through neutral resistor is same as that of fault current. ie $\underline{\underline{3.07 \angle -14.9^\circ \text{ kA}}}$

(iii) Voltage across the grounding Resistor.

$$= I_f \cdot R_n$$

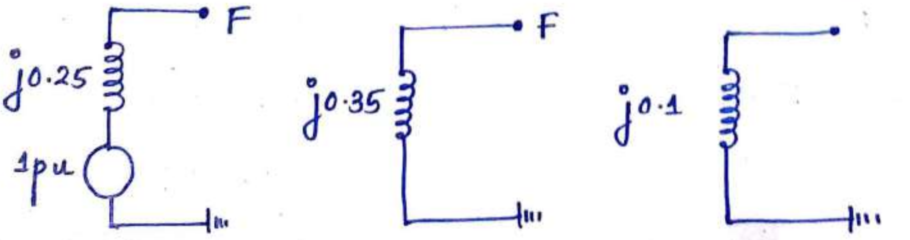
$$= 3.07 \angle -14.9^\circ \times 2$$

$$= \underline{\underline{6.14 \angle -14.9^\circ \text{ kV}}}$$

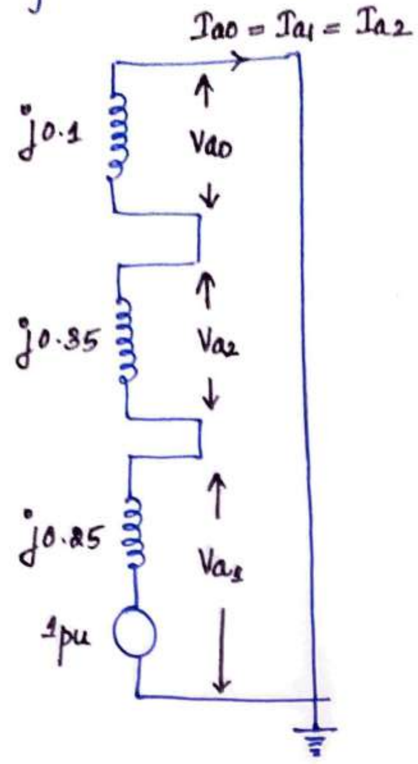
2. A salient - pole generator without dampers is rated 20 MVA, 13.8 kV and has a direct axis subtransient reactance of 0.25 per unit. The negative and zero sequence reactance are 0.35 and 0.1 p.u respectively. The neutral of the generator is solidly grounded. Determine the sub-transient current in the generator and the line to line voltages for subtransient conditions when a S.L.G fault occurs at the generator terminals. with generator operating unloaded at rated voltage. Neglect resistance.

sol$^n$. Choosing $MVA_b = 20$ and $kV_b = 13.8$

$X_{g,1} = 0.25j$, $X_{g,2} = 0.35j$ $X_{g,0} = 0.1j$ p.u    Y⏚ , SLG fault.



$\therefore$ for SLG




Subtransient current in the generator is the fault current $I_a = 3.I_{a1}$.

$$I_{a1} = \frac{1}{j0.25 + j0.35 + j0.1}$$

$$= \underline{\underline{1.428 \angle -90^\circ \ p.u}}$$

$$\therefore I_a = 3.I_{a1}$$

$$= 3 \times 1.428 \angle -90^\circ$$

$$= \underline{\underline{4.285 \angle -90^\circ \ p.u}}$$

$\therefore$ Actual Value $=$ p.u value $\times$ Base Value.

$$\text{Base Current} = \frac{kVA_b}{\sqrt{3} \times kV_b} = \frac{20 \times 1000}{13.8 \times \sqrt{3}}$$

$$= 836.739 \ A$$

$\therefore$ Actual value of subtransient current $= \underline{\underline{3585.42 \ A}}$

To find line to line Voltages;

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$V_{a0} = -I_{a0}.X_0$$

$$= -1.428 \angle -90^\circ \times j0.1$$

$$= \underline{\underline{-0.1428 \ p.u}}$$

$$V_{a1} = 1 - I_{a1}.X_1$$

$$= 1 - 1.428 \angle -90 \times j0.25$$

$$= \underline{\underline{0.643 \ p.u}}$$

$$V_{a2} = -I_{a2}.Z_2$$

$$= -1.428 \times j0.35$$

$$= \underline{\underline{-0.4998 \ p.u}}$$

$V_a = V_{a0} + V_{a1} + V_{a2}$ Since in SLG $V_a = 0$

$= \underline{\underline{0\,V}}$ since $Z_f = 0$.

$V_b = V_{a0} + \alpha^2 V_{a1} + \alpha V_{a2}$

$= -0.1428 + (1\angle 240)\,0.643 - (1\angle 120)(0.4998)$

$= \underline{\underline{1.0126\angle -102^\circ\ p.u}}$

$V_c = V_{a0} + \alpha V_{a1} + \alpha^2 V_{a2}$

$= -0.1428 + (1\angle 120)\,0.643 - (1\angle 240)(0.4998)$

$= \underline{\underline{1.0126\angle 102^\circ\ p.u}}$

∴ Line voltages;

$V_{ab} = V_a - V_b$

$= \underline{\underline{1.0126\angle 78^\circ\ p.u}}$

$V_{bc} = V_b - V_c$

$= \underline{\underline{1.99\angle -90^\circ\ p.u}}$

$V_{ca} =$

$\underline{\underline{1.0126\angle 102^\circ}}$

To find actual value :- p.u value × base value ; Base Value (phase) $= \frac{13.8}{\sqrt{3}}$

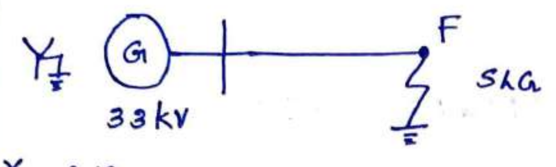
$V_{ab} = 1.0126\angle 78^\circ \times \frac{13.8}{\sqrt{3}} = \underline{\underline{8.06\angle 78^\circ\ kV}}$

$V_{bc} = \underline{\underline{15.85\angle -90^\circ\ kV}}$

$V_{ca} = \underline{\underline{8.06\angle 102^\circ\ kV}}$

3. A three phase, 37.5 MVA, 33 kV alternator having $X_1 = 0.$[illegible] $X_2 = 0.12$ pu and $X_0 = 0.1$ p.u, based on its rating is connected to a 33[illegible] overhead line having $X_1 = 6.3\,\Omega$ and $X_2 = 6.3\,\Omega$, $X_0 = 12.6\,\Omega$ per phase. A SLG fault occurs at the remote end of the line. The alternator neutral is solidly grounded. Calculate the fault current (University. Q Nov – 2015 8 marks)

Solⁿ.




$X_1 = 0.18$

$X_2 = 0.12$

$X_0 = 0.1$

Choose Base of 37.5 MVA, 33 kV

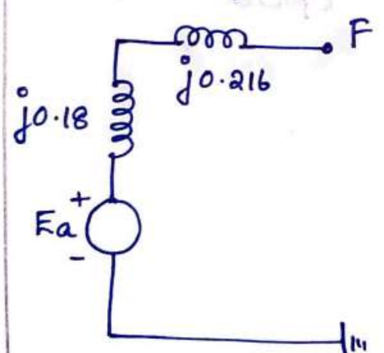
$$\therefore Z_b = \frac{(kV_b)^2}{MVA_b} = \frac{33^2}{37.5}$$

$$= \underline{29.04\,\Omega}$$

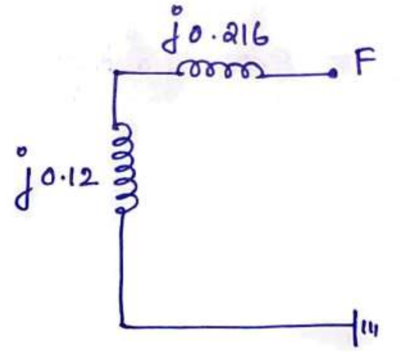
Transmⁿ line :-

$$X_1 = \frac{6.3}{29.04} = j0.216 \text{ p.u} \;;\; X_2 = j0.216 \text{ p.u} \;;\; X_0 = \frac{12.6}{29.04} = j0.433 \text{ p.u}$$

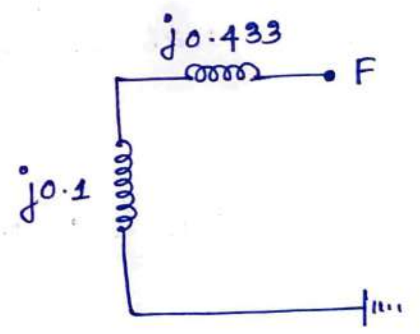
∴ +ve Sequence :



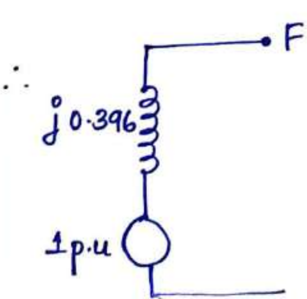

–ve Sequence

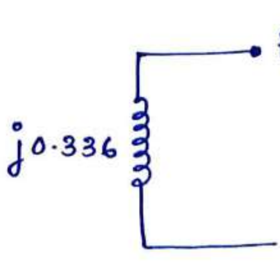


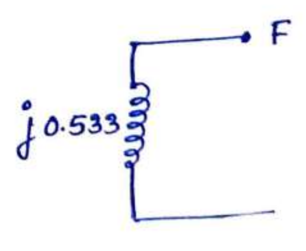

Zero Sequence

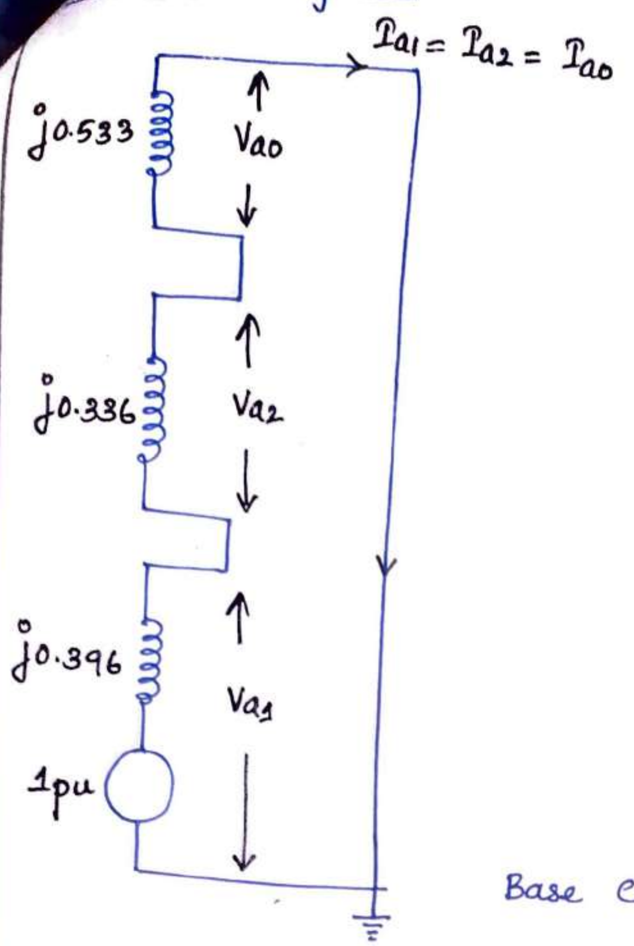



∴






Fault current $= I_a = 3.I_{a1}$

$$I_{a1} = \frac{1}{j0.396 + j0.336 + j0.533}$$

$$= 0.7905 \angle -90^\circ \text{ p.u}$$

$$\therefore \quad I_f = 3 \times 0.7905 \angle -90^\circ$$

$$= \underline{\underline{2.371 \angle -90^\circ \text{ p.u}}}$$

Base Current $= \dfrac{kVA_b}{\sqrt{3}\ kV_b} = \dfrac{37.5 \times 1000}{\sqrt{3} \times 33}$

$$= 656.07 \text{ A}.$$

$\therefore$ Actual value of fault current $= 2.371 \angle -90^\circ \times 656.07$

$$= \underline{\underline{1.55 \angle -90^\circ \text{ kA}}}$$

4\. A S.L.G fault of 0.05 Ω resistance occur in a 3ϕ system supplied by a synchronous generator with a generated emf of 11kV between the lines. The +ve, -ve and zero sequence reactance of the generator and network upto the fault are 0.5 Ω, 0.2 Ω and 0.1 Ω respy. Find the fault current.

(University May - 2016 10 marks)

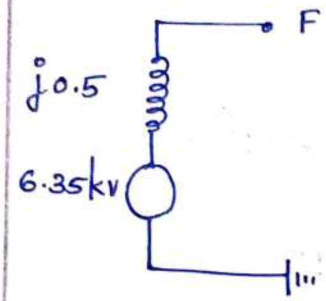
Sol: Here actual values are given;

$Z_f = 0.05\,\Omega$ (Resistance) kv line = 11 kv

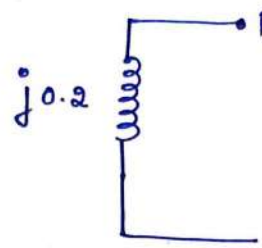
$\therefore$ phase = 6.35 kV.

$X_1 = j0.5\,\Omega$, $X_2 = j0.2\,\Omega$, $X_0 = j0.1\,\Omega$.

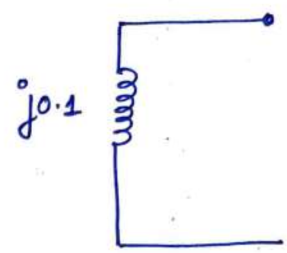
+ve Sequence :



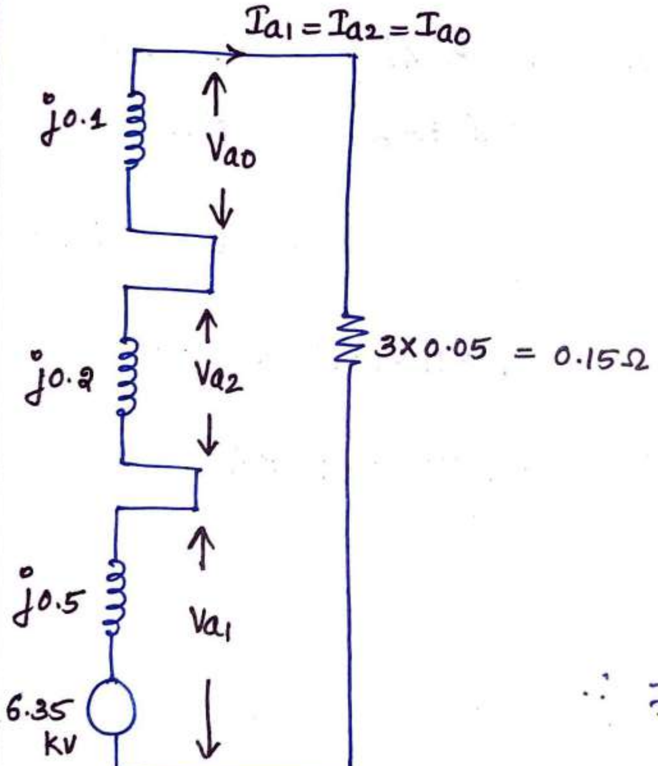

-ve Sequence




Zero Sequence




$\therefore$ for SLG




Fault Current $= 3 \cdot I_{a1}$

$$I_{a1} = \frac{6.35\text{ kV}}{j0.5 + j0.2 + j0.1 + 0.15}$$

$$= 7.801 \angle -79.3^\circ \text{ kA}$$

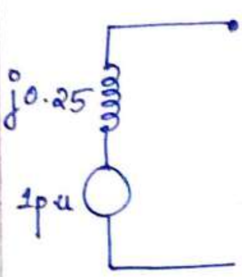
$\because$ Fault Current $= 7.801 \angle -79.3^\circ \times 3$

$$= 23.40 \angle -79.3^\circ \text{ kA}$$

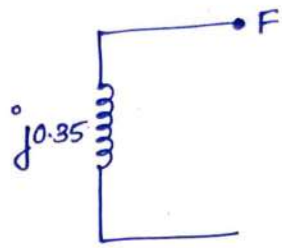
Qn no: (2) Find the subtransient currents and line to line voltages at the fault when a line to line fault between the phases b and c occurs. Assume the generator is unloaded and operating at rated terminal voltage when fault occurs. Neglect resistance.

sol: From Qn no: (2)

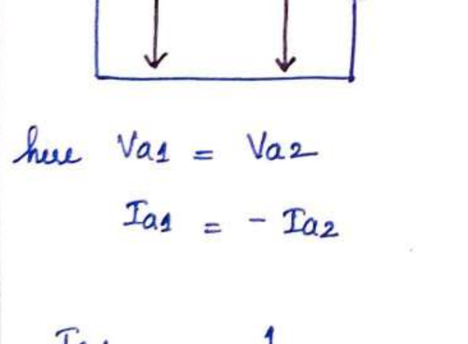
+ve sequence:




-ve sequence:




∴ For L-L fault;




here $V_{a1} = V_{a2}$

$I_{a1} = -I_{a2}$

$$I_{a1} = \frac{1}{j0.25 + j0.35}$$

$= 1.667 \angle -90^\circ$ p.u

Fault Current $= I_b$

$= I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2}$

But $I_{a0} = 0$

$\therefore I_b = (\alpha^2 - \alpha) \cdot I_{a1}$

Since $I_{a1} = -I_{a2}$.

$\Rightarrow \quad \therefore I_b = (1\angle 240 - 1\angle 120)\, 1.667\angle -90^\circ$

$= 2.887 \angle 180^\circ$ p.u

Actual value of fault current $= 2.887\angle 180^\circ \times \underbrace{836.7}_{\text{Base Current, from Qn no: 2}}$

$= 2.416$ kA $\angle 180^\circ$

Similarly calculate the line voltages $V_a$, $V_b$ and $V_c$.

here $V_{a0} = 0$; $V_{a1} = V_{a2} = E - I_{a1}.Z_1$

$$= 1 - (1.667\angle -90^\circ)(j0.25)$$

$$= \underline{\underline{0.583\ p.u}}$$

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$= 1.166\angle 0^\circ\ p.u = \underline{\underline{9.29\ kV}} \quad \left\{ \text{since Base value} = 13.8/\sqrt{3} \right\}$$

$$V_b = V_{a0} + \alpha^2 V_{a1} + \alpha V_{a2} = V_c$$

$$= (\alpha^2 + \alpha).V_{a1} = (1\angle 240 + 1\angle 120).0.583$$

$$= \underline{\underline{0.583 p.u}}\angle 180^\circ = \underline{\underline{4.645\ kV}}\angle 180^\circ.$$

$$\therefore V_{ab} = 9.29 - 4.645\angle 180^\circ.$$

$$= \underline{\underline{13.93\ kV}}$$

$$V_{bc} = 4.645\angle 180^\circ - 4.645\angle 180^\circ$$

$$= \underline{\underline{0}}$$

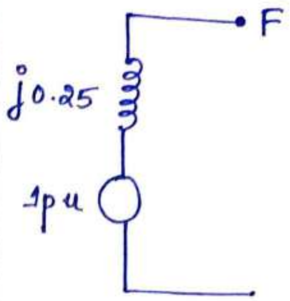
$$V_{ca} = 4.645\angle 180^\circ - 9.29$$

$$= \underline{\underline{13.93\angle 180^\circ\ kV}}$$

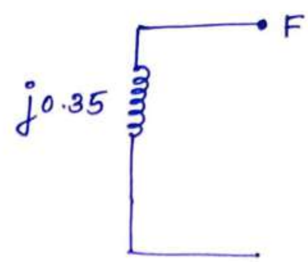
25 MVA, 13.2 kV alternator with solidly grounded neutral has a subtransient reactance of 0.25 p.u. Negative and zero sequence reactance are 0.35 p.u and 0.1 p.u respy. A L-L fault occurs at the terminals of an unloaded generator. Determine fault current and L-L voltages. Neglect resistances. (University - May 2015 10 marks).

sol: Choose base values 25 MVA, 13.2 kV

+ve sequence



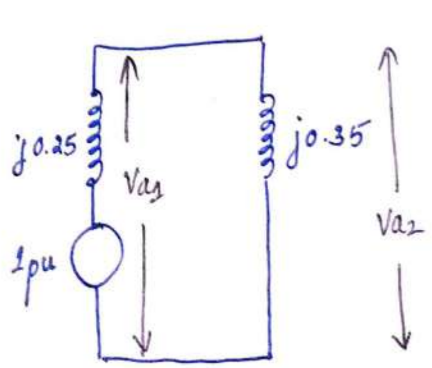

-ve sequence




Solution same as the previous Question; only change is the base values of MVA and kV.

$$\therefore \quad \text{Base Current} = \frac{kVA_b}{\sqrt{3}\,.\,kV_b} = \frac{25 \times 1000}{\sqrt{3} \times 13.2} = \underline{\underline{1093.46\,A}}$$

$$\text{Base kV for phase voltage} = \frac{13.2}{\sqrt{3}} = \underline{\underline{7.62\,kV}}$$




Fault current $I_f = I_b$

$= (\alpha^2 - \alpha)\, I_{a1}$

$$I_{a1} = \frac{1}{j0.25 + j0.35} = 1.66 \angle -1.57°$$

$\therefore$ I

# Load Flow Analysis – Newton Raphson Method.

## Mathematical Background

Complex $S_i = P_i + jQ_i$

$= v_i . I_i^*$ ;

Injected current $I_i = \sum_{q=1}^{n} Y_{iq} . V_q$ where $i = i^{th}$ bus, $n$ = total no: of buses.

In polar form;

$Y_{iq} = |Y_{iq}| \angle \theta_{iq}$

$v_q = |v_q| \angle \delta_q$

$v_i = |v_i| \angle \delta_i$

$\therefore$ Complex power; $S_i = P_i + jQ_i = v_i . I_i^*$.

$$\therefore P_i + jQ_i = |v_i| \angle \delta_i \left[ \sum_{q=1}^{n} Y_{iq} . V_q \right]^*$$

$$= |v_i| \angle \delta_i \left[ \sum_{q=1}^{n} |Y_{iq}| \angle -\theta_{iq} . |v_q| \angle -\delta_q \right]$$

$$= |v_i| \sum_{q=1}^{n} |Y_{iq}| |v_q| \angle \delta_i - \delta_q - \theta_{iq}.$$

$\therefore P_i$ = Real part of the Equation

$$= |v_i| \sum_{q=1}^{n} |Y_{iq}| |V_q| \cos \angle \delta_i - \delta_q - \theta_{iq}$$

$\therefore Q_i$ = Imaginary part.

$$= |v_i| \sum_{q=1}^{n} |Y_{iq}| |V_q| \sin \angle \delta_i - \delta_q - \theta_{iq}.$$

From Equations of $P_i$ & $Q_i$; we can write $P_i = f(|V|, \delta)$
$Q_i = f(|V|, \delta)$.

Let $y = f(x)$

As per Taylor Series $$y = f(x_0) + \left.\frac{\partial f}{\partial x}\right|_{x=x_0} (x - x_0) + \cdots\cdots$$

$$\therefore\ y - f(x_0) = \left.\frac{\partial f}{\partial x}\right|_{x=x_0} (x - x_0)$$

$$= \frac{\partial f}{\partial x} \cdot x - \frac{\partial f}{\partial x} \cdot x_0$$

$$\therefore\ \left[y - f(x_0)\right] + \frac{\partial f}{\partial x} \cdot x_0 = \frac{\partial f}{\partial x} \cdot x.$$

or. $$\left[\frac{\partial f}{\partial x}\right]^{-1} (y - f(x_0)) + x_0 = x$$

or $$x = x_0 + \left[\frac{\partial f}{\partial x}\right]^{-1} (y - f(x_0))$$

or $$\boxed{x^{k+1} = x^{k} + \left[\frac{\partial f}{\partial x}\right]^{-1} (y - f(x^{k}))}$$

Back to power System.

$P = f(V, \delta)$
$Q = f(V, \delta)$

$y = f(x)$ ; function ; Variables

; here 2 funⁿ P & Q
2 Variables $V$ & $\delta$

$\therefore$ $x \to$ Variables are $V$ & $\delta$.

$f \to$ Functions are $P$ & $Q$.

Consider a s/m with total no: of buses = $n$ & Bus no: 1 is slack.

$\therefore$ sets of variables are $\delta_2, \delta_3, \delta_4 \ldots\ldots, \delta_n$ & $|V_2|, |V_3|, \ldots\ldots |V_n|$.

$\therefore$ Refering to our main eq$^n$;

$$x^{k+1} = x^k + \left[\frac{\partial f}{\partial x}\right]^{-1}(y - f(x^k))$$

$$\begin{bmatrix} \delta_2^{k+1} \\ \delta_3^{k+1} \\ \vdots \\ \delta_n^{k+1} \\ |V_2|^{k+1} \\ |V_3|^{k+1} \\ \vdots \\ |V_n|^{k+1} \end{bmatrix} = \begin{bmatrix} \delta_2^{k} \\ \delta_3^{k} \\ \vdots \\ \delta_n^{k} \\ |V_2|^{k} \\ |V_3|^{k} \\ \vdots \\ |V_n|^{k} \end{bmatrix} + \underbrace{\left[\frac{\partial f}{\partial x}\right]^{-1}}_{\text{Jacobian matrix } (J)} \begin{bmatrix} P_2^{k+1} - P_2^k \\ P_3^{k+1} - P_3^k \\ \vdots \\ P_n^{k+1} - P_n^k \\ Q_2^{k+1} - Q_2^k \\ Q_3^{k+1} - Q_3^k \\ \vdots \\ Q_n^{k+1} - Q_n^k \end{bmatrix}$$

$$J = \left[\begin{array}{c|c} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial |V|} \\ \hline \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial |V|} \end{array}\right]$$

$$\begin{array}{cc} J_1 & J_2 \end{array}$$

$$J = \left[\begin{array}{cccc|cccc} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \cdots & \frac{\partial P_2}{\partial \delta_n} & \frac{\partial P_2}{|V_2|} & \frac{\partial P_2}{\partial |V_3|} & \cdots & \frac{\partial P_2}{\partial |V_n|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \cdots & \frac{\partial P_3}{\partial \delta_n} & \frac{\partial P_3}{\partial |V_2|} & \frac{\partial P_3}{\partial |V_3|} & \cdots & \frac{\partial P_3}{\partial |V_n|} \\ \vdots & & & & & & & \vdots \\ \frac{\partial P_n}{\partial \delta_2} & \frac{\partial P_n}{\partial \delta_3} & \cdots & \frac{\partial P_n}{\partial \delta_n} & \frac{\partial P_n}{\partial |V_2|} & \frac{\partial P_n}{\partial |V_3|} & \cdots & \frac{\partial P_n}{\partial |V_n|} \\ \hline \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \cdots & \frac{\partial Q_2}{\partial \delta_n} & \frac{\partial Q_2}{\partial |V_2|} & \frac{\partial Q_2}{\partial |V_3|} & \cdots & \frac{\partial Q_2}{\partial |V_n|} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \cdots & \frac{\partial Q_3}{\partial \delta_n} & \frac{\partial Q_3}{\partial |V_2|} & \frac{\partial Q_3}{\partial |V_3|} & \cdots & \frac{\partial Q_3}{\partial |V_n|} \\ \vdots & & & & & & & \\ \frac{\partial Q_n}{\partial \delta_2} & \frac{\partial Q_n}{\partial \delta_3} & \cdots & \frac{\partial Q_n}{\partial \delta_n} & \frac{\partial Q_n}{\partial |V_2|} & \frac{\partial Q_n}{\partial |V_3|} & \cdots & \frac{\partial Q_n}{\partial |V_n|} \end{array}\right]$$

$$\begin{array}{cc} J_3 & J_4 \end{array}$$

$$\therefore \begin{bmatrix} \delta^{k+1} \\ |V|^{k+1} \end{bmatrix} = \begin{bmatrix} \delta^{k} \\ |V|^{k} \end{bmatrix} + \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} P^{k+1} - P^{k} \\ Q^{k+1} - Q^{k} \end{bmatrix}$$

$$J_1 = \frac{\partial P_i}{\partial \delta_q} = \frac{\partial}{\partial \delta_q} \left\{ |V_i| \sum_{q=1}^{n} |Y_{iq}| |V_q| \cos(\delta_i - \delta_q - \theta_{iq}) \right\}$$

$$= |V_i| \sum_{q=1}^{n} |Y_{iq}| |V_q| \sin(\delta_i - \delta_q - \theta_{iq}) \}$$

$$J_2 = \frac{\partial P_i}{\partial |V_q|} = \frac{\partial}{\partial V_q} \left\{ |V_i| \sum_{q=1}^{n} |Y_{iq}| |V_q| \cos(\delta_i - \delta_q - \theta_{iq}) \right\}$$

$$= |V_i| \sum_{q=1}^{n} |Y_{iq}| \cos(\delta_i - \delta_q - \theta_{iq})$$

$$J_3 = \frac{\partial Q_i}{\partial \delta_q} = \frac{\partial}{\partial q} \left\{ |V_i| \sum_{q=1}^{n} |Y_{iq}| |V_q| \sin(\delta_i - \delta_q - \theta_{iq}) \right\}$$

$$= -|V_i| \sum_{q=1}^{n} |Y_{iq}| |V_q| \cos(\delta_i - \delta_q - \theta_{iq})$$

$$\text{lly} \quad J_4 = \frac{\partial Q_i}{\partial |V_q|} = |V_i| \sum_{q=1}^{n} |Y_{iq}| \sin(\delta_i - \delta_q - \theta_{iq}).$$

$$\therefore \quad J = \left[\begin{array}{c|c} J_4 . |V_q| & J_2 \\ \hline -|V_q| \, J_2 & J_4 \end{array}\right]$$

# Load Flow Analysis – De-Coupled Load flow.

Assumptions in the Jacobian matrix.

- Real power P mainly dependent on angle of voltage 'δ', than its magnitude

$$\therefore \frac{\partial P}{\partial V_q} = 0$$

- Reactive power Q mainly depends on magnitude of voltage, than its Angle.

$$\therefore \frac{\partial Q}{\partial \delta_q} = 0$$

$$\therefore \text{Jacobian } J = \begin{bmatrix} \frac{\partial P_i}{\partial \delta_q} & 0 \\ 0 & \frac{\partial Q_i}{\partial V_q} \end{bmatrix}$$

i.e.
$$\begin{bmatrix} \delta^{k+1} \\ |V|^{k+1} \end{bmatrix} = \begin{bmatrix} \delta^{k} \\ |V|^{k} \end{bmatrix} + \begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix}^{-1} \begin{bmatrix} P^{k+1} - P^{k} \\ Q^{k+1} - Q^{k} \end{bmatrix}$$

or
$$\begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

where $\Delta\delta = \delta^{k+} - \delta^{k}$

$\Delta V = |V|^{k+1} - |V|^{k}$

$\Delta P = P^{k+1} - P^{k}$

$\Delta Q = Q^{k+1} - Q^{k}$.

# Fast Decoupled Load Flow Analysis (FDLF)

We know;

$$\begin{bmatrix} \Delta \delta \\ \Delta v \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

we also know;

$$J_1 = V_i \sum_{q=1}^{n} Y_{iq} V_q \sin(\delta_i - \delta_q - \theta_{iq})$$

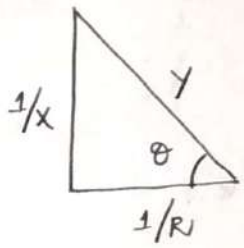
$$J_4 = V_i \sum_{1}^{n} Y_{iq} . \sin(\delta_i - \delta_q - \theta_{iq})$$

**Assumptions :-**

- Angle of voltage in 'p.u' will not differ much ; hence $\delta_i - \delta_q \simeq 0$
- Voltage of indiv. bus will be approx equal to 1. $\therefore$ $V_q \simeq 1$.

$$\therefore \quad J_1 = V_i \sum_{q=1}^{n} Y_{iq} . \sin(-\theta_{iq})$$

$$J_4 = V_i \sum_{q=1}^{n} \underbrace{Y_{iq} . \sin(-\theta_{iq})}_{\longrightarrow\, B_{iq}.}$$




$\therefore$ $Y \sin\theta = \frac{1}{X} = B$

$$\therefore \quad J_1 = V_i \sum_{q=1}^{n} - B_{iq}$$

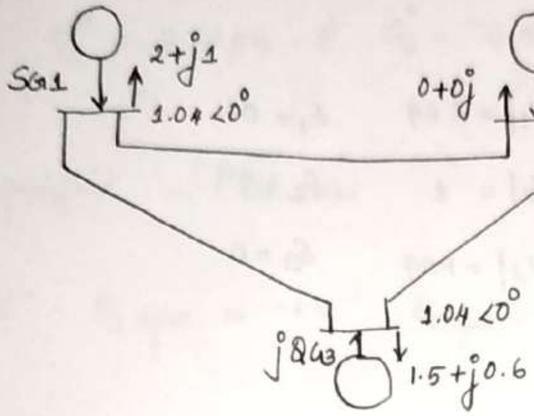
$$J_4 = V_i \sum_{q=1}^{n} - B_{iq}.$$

$$\therefore \begin{bmatrix} \Delta \delta \\ \Delta v \end{bmatrix} = \begin{bmatrix} -B_{iq} . V_i & 0 \\ 0 & -B_{iq} V_i \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} -B_{iq} . V_i & 0 \\ 0 & -B_{iq} V_i \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta v \end{bmatrix}$$

$$\begin{bmatrix} \frac{\Delta P}{|V_i|} \\ \frac{\Delta Q}{|V_i|} \end{bmatrix} = \begin{bmatrix} -B_{iq} . & 0 \\ 0 & -B_{iq} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta v \end{bmatrix}$$

# Problem : N.R Method.

Q) Consider the three bus system, shown in figure, Each of the three lines has a series impedance of $0.02 + j0.08$ and a total shunt admittance of $j0.02$pu. The Specific quantities at the buses are tabulated below.

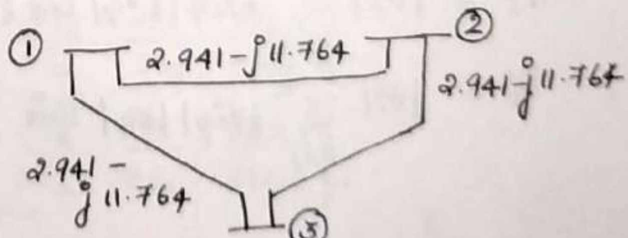



| Bus | Real load demand | Reactive load demand | Real power genern | Reactive power genern | Voltage spec. |
|---|---|---|---|---|---|
| 1 | 2.0 | 1.0 | Unspec. | Unspec | $V_1 = 1.04 + j0$ |
| 2 | 0.0 | 0.0 | 0.5 | 1.0 | Unspec. |
| 3 | 1.5 | 0.6 | 0.0 | ? | $\lvert V_3 \rvert = 1.04$<br>δ not spec. |

Controllable reactive power source is available at bus 3 with constraint

$$0 \le Q_{G_3} \le 1.5 \text{ p.u.}$$

Solⁿ: Step 1: Calculate Ybus.




Series imp $= 0.02 + j0.08$

$$\therefore \text{Adm} = \frac{1}{0.02 + j0.08} = 2.941 - j11.764$$

Total Shunt Admitt $= j0.02$.

$$\therefore \text{All off diagonal} = -(2.941 - j11.764)$$

$$= -2.941 + j11.764$$

Diagonal Elements

$$Y_{11} = Y_{22} = Y_{33}$$

$$= 2.941 - j11.764 + 2.941 - j11.764 + j0.02$$

$$= 5.882 - j23.528$$

$$Y_{bus} = \begin{bmatrix} 24.23\angle -75.95^\circ & 12.13\angle 104.04^\circ & 12.13\angle 104.04^\circ \\ 12.13\angle 104.04^\circ & 24.23\angle -75.95^\circ & 12.13\angle 104.04^\circ \\ 12.13\angle 104.04^\circ & 12.13\angle 104.04^\circ & 24.23\angle -75.95^\circ \end{bmatrix}$$

Step 2: Initialize bus voltages

$V_1^0 = 1.04\angle 0^\circ$ ie; $|V_1| = 1.04$ $\delta_1 = 0$

$V_2^0 = 1\angle 0^\circ$ $|V_2| = 1$ $\delta_2 = 0$

$V_3^0 = 1.04\angle 0^\circ$ $|V_3| = 1.04$ $\delta_3 = 0$

Refer our basic Equation.

$$\begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

here the $\delta$'s are Unknown $\delta$ & $|V|$ are Unknown $|V|$,

P's are Specified (known) P & Q's are Specified (known) Qs.

$\therefore$ here Specified P's are $P_2$ & $P_3$

Specified Q's are $Q_2$ only.

we know;

$$P_i^o = |V_i^o| \sum_{q=1}^{n} |Y_{iq}||V_q| \cos(\delta_i - \delta_q - \theta_{iq})$$

$$Q_i^o = |V_i^o| \sum_{q=1}^{n} |Y_{iq}||V_q| \sin(\delta_i^o - \delta_q - \theta_{iq})$$

$$\therefore P_2 = |V_2| \sum_{q=1}^{3} |Y_{iq}||V_q| \cos(\delta_i^o - \delta_q - \theta_{iq}^o)$$

$$= |V_2| \left\{ Y_{21} V_1 \cos(\delta_2 - \delta_1 - \theta_{12}) + Y_{22} V_2 \cos(\delta_2 - \delta_2 - \theta_{22}) + Y_{23} V_3 \cos(\delta_2 - \delta_3 - \theta_{23}) \right\}$$

lly $P_3 = |V_3| \left\{ Y_{31} V_1 \cos(\delta_3 - \delta_1 - \theta_{13}) + Y_{32} V_2 \cos(\delta_3 - \delta_2 - \theta_{23}) + Y_{33} V_3 \cos(\delta_3 - \delta_3 - \theta_{23}) \right\}$

$Q_2 = |V_2| \left\{ Y_{21} V_1 \sin(\delta_2 - \delta_1 - \theta_{13}) + Y_{22} V_2 \sin(\delta_2 - \delta_2 - \theta_{22}) + Y_{23} V_3 \sin(\delta_2 - \delta_3 - \theta_{23}) \right\}$

Given values are; substituted ; Assumed values are also substituted

$$P_2^0 = 1 \left\{ (12.13)(1.04) \cdot \cos(0-0-104.04) + 24.23 \times 1 \times \cos(0-0+75.95) + (12.13 \times 1.04)(\cos(0-0-1.04.04) \right\}$$

$$= \underline{\underline{-0.23 \, p.u}}$$

lly $P_3^0 = 0.12 \, pu$ & $Q_2^0 = -0.96 \, p.u$

$\Delta P = P_{specified} - P_{calculated}$

$P_2 \, spec = 0.5$ $P_3 \, spec = -1.5$ $Q_2 spec = 1$.

$\therefore \Delta P_2^0 = P_2 \, spec - P_2^0$ ; $\Delta P_3^0 = P_3 \, spec - P_3^0$

$= 0.5 - (-0.23) = \underline{\underline{0.73}}$ $= -1.5 - 0.12 = \underline{\underline{-1.62}}$

$\Delta Q_2^0 = Q_2 \, spec - Q_2^0$

$= 1 - (-0.96) = \underline{\underline{1.96}}$

$$\therefore \begin{bmatrix} \Delta P_2^0 \\ \Delta P_3^0 \\ \Delta Q_2^0 \end{bmatrix} = \begin{bmatrix} 0.73 \\ -1.62 \\ 1.96 \end{bmatrix}$$

Now; to find Jacobian Elements

$$\left[\begin{array}{cc|c} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} \\ \hline \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} \end{array}\right]$$

We know;

$$P_2 = V_2 \cdot Y_{21} V_1 \cos(\delta_2 - \delta_1 - \theta_{12}) + |V_2|^2 Y_{22} \cos(\delta_2 - \delta_2 - \theta_{22}) + V_2 \cdot Y_{23} \cdot V_3 \cos(\delta_2 - \delta_3 - \theta_{23})$$

$$\therefore \frac{\partial P_2}{\partial \delta_2} = -V_2 Y_{21} V_1 \sin(\delta_2 - \delta_1 - \theta_{12}) + 0 - V_2 Y_{23} V_3 \cdot \sin(\delta_2 - \delta_3 - \theta_{23})$$

$$= -1 \times 12.13 \times 1.04 \sin(0-0-1.04.04) + 0 - 1 \times 12.13 \times 1.04 (0-0-1.04.04)$$

$$= 24.47 //$$

Ily $\dfrac{\partial P_2}{\partial \delta_3} = 0 + 0 + V_2 \cdot Y_{23} \cdot V_3 \sin(\delta_2 - \delta_3 - \theta_{23})$

$= 1 \times 12.13 \times 1.04 \sin(0 - 0 - 104.04^\circ) = \underline{-12.23}$

$\dfrac{\partial P_2}{\partial V_2} = Y_{21} \cdot V_1 \cos(\delta_2 - \delta_1 - \theta_{12}) + 2 V_2 \cdot Y_{22} \cos(\delta_2 - \delta_2 - \theta_{22}) + V_3 \cdot Y_{23} \cos(\delta_2 - \delta_3 - \theta_{23})$

$= \underline{5.64}$

Ily calculate all Jacobian elements, & we will obtain $J = \begin{bmatrix} 24.47 & -12.23 & 5.64 \\ -12.23 & 24.95 & -3.05 \\ -6.11 & 3.05 & 22.54 \end{bmatrix}$

$$\therefore \begin{bmatrix} \Delta \delta_2^1 \\ \Delta \delta_3^1 \\ \Delta |V_2|^1 \end{bmatrix} = \begin{bmatrix} 24.47 & -12.23 & 5.64 \\ -12.23 & 24.95 & -3.05 \\ -6.11 & 3.05 & 22.54 \end{bmatrix}^{-1} \begin{bmatrix} 0.73 \\ -1.62 \\ 1.96 \end{bmatrix}$$

$$= \begin{bmatrix} -0.023 \\ -0.0654 \\ 0.089 \end{bmatrix}$$

$\therefore \delta_2^1 = \delta_2^0 + \Delta \delta_2^1$

$= 0 + -0.023 = -0.023$

$\delta_3^1 = \delta_3^0 + \Delta \delta_3^1$

$= 0 + -0.0654 = -0.0654$

$|V_2|^1 = |V_2^0| + \Delta |V_2|^1$

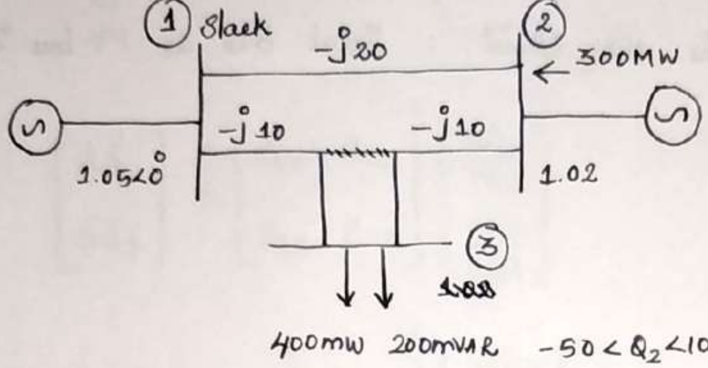
$= 1 + 0.089 = 1.089$

$\therefore$ $V_2^1 = 1.089 \angle -0.023^\circ$

$V_3^1 = 1.04 \angle -0.0654^\circ$

Problem : Fast De Coupled Load flow.

Q) Obtain the power flow solution (one iteration) for the system shown in figure. The line admittance are in per unit on a 100 MVA base. Use FDLF.




$sol^n$:

Step 1: Calculate Y bus.

$$Y_{bus} = \begin{bmatrix} -j30 & j20 & j10 \\ j20 & -j30 & j10 \\ j10 & j10 & -j20 \end{bmatrix} = \begin{bmatrix} 30\angle -1.57 & 20\angle 1.57 & 10\angle 1.57 \\ 20\angle 1.57 & 30\angle -1.57 & 10\angle 1.57 \\ 10\angle 1.57 & 10\angle 1.57 & 20\angle -1.57 \end{bmatrix}$$

Step 2: Initialize Bus voltages

$V_1^0 = 1.05\angle 0^\circ$ slack ; $V_2^0 = 1.02\angle 0^\circ$ ; $V_3^0 = 1\angle 0^\circ$ (PQ Bus).

Step 3: check for $Q_2$ limit violation

$$Q_2 = \Big\{ |V_2||V_1||Y_{12}| \sin(\delta_2 - \delta_1 - \theta_{12}) + |V_2||V_2||Y_{22}| \sin(\delta_2 - \delta_2 - \theta_{22}) + |V_2||V_3||Y_{23}| \sin(\delta_2 - \delta_3 - \theta_{23}) \Big\}$$

$$= \Big\{ 1.02 \times 1.05 \times 20 . \sin(0 - 0 - 1.57) + 1.02 \times 1.02 \times 30 . \sin(0 - 0 + 1.57) + 1.02 \times 1 \times 10 \sin(0 - 0 - 1.57) \Big\} = \underline{-0.408\,pu}$$

Limit given is ; $-50 \leq Q_2 < 100$ MVAR ; Take base as 100

$$\therefore \text{limit} = \frac{-50}{100} < Q_2 < \frac{100}{100} \text{ p.u}$$

$$= -0.5 < Q_2 < 1 \text{ p.u}$$

$Q_2$ value satisfies the limit. ; Treat bus as PV bus itself.

Step 4: Calculate $\Delta P$ & $\Delta Q$.

$$\Delta P = P_{spec} - P_{cal}$$

$$P_2 \text{ Spec} = P_{G_2} - P_{L_2} = 300 - 0 = \frac{300}{100} = 3 \text{ pu}$$

$$P_3 \text{ Spec} = P_{G_3} - P_{L_3} = 0 - 400 = \frac{-400}{100} = -4 \text{ pu}$$

$$Q_3 \text{ Spec} = Q_{G_3} - Q_{L_3} = 0 - 200 = \frac{-200}{100} = -2 \text{ pu}$$

$$P_2 \text{ cal} = |V_2||V_1||Y_{12}| \cos(\delta_2 - \delta_1 - \theta_{12}) + |V_2||V_2| Y_{22} \cos(\delta_2 - \delta_2 - \theta_{22}) + |V_2||V_3| Y_{23} \cos(\delta_2 - \delta_3 - \theta_{23})$$

$$= 1.02 \times 1.05 \times 20 \cos(0 - 0 - 1.57) + 1.02 \times 1.02 \times 30 \times \cos(0 - 0 + 1.57) + 1.02 \times 1 \times 20 (0 - 0 \pm 1.57)$$

$$= \underline{\underline{0.05 \text{ p.u}}}$$

lly $P_3$ cal $= \underline{\underline{0.0324}}$

$Q_3$ cal $= \underline{\underline{-0.7}}$

$$\therefore \Delta P_2 = 3 - 0.05 = 2.95$$

$$\Delta P_3 = -4 - 0.0324 = 4.0324$$

$$\Delta Q_3 = -2 + 0.7 = -1.3$$

## Step 5: Bus Susceptance Matrix.

we know;

$$\begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = -\begin{bmatrix} B' & 0 \\ 0 & B'' \end{bmatrix}^{-1} \begin{bmatrix} \frac{\Delta P}{|V|} \\ \frac{\Delta Q}{|V|} \end{bmatrix}$$

ie $\Delta\delta = -[B']^{-1}\left[\frac{\Delta P}{V}\right]$

$\Delta V = -[B'']^{-1}\left[\frac{\Delta Q}{V}\right]$

$\therefore$ here

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \end{bmatrix} = -\begin{bmatrix} B_{22} & B_{23} \\ B_{32} & B_{33} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\Delta P_2}{|V_2|} \\ \frac{\Delta P_3}{|V_3|} \end{bmatrix}$$

&

$$[\Delta V_3] = -[B_{33}]^{-1}\left[\frac{\Delta Q_3}{|V_3|}\right]$$

$$B' = \begin{bmatrix} -30 & 10 \\ 10 & -20 \end{bmatrix} \qquad B'' = [-20]$$

## Step 6: Calculate $\Delta\delta$ & $\Delta V$

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \end{bmatrix} = -\begin{bmatrix} -30 & 10 \\ 10 & -20 \end{bmatrix}^{-1} \begin{bmatrix} \frac{2.95}{1.02} \\ -\frac{4.0324}{1} \end{bmatrix} = \begin{bmatrix} 0.035 \\ -0.184 \end{bmatrix}$$

$\therefore \delta_2^1 = \delta_2^0 + \Delta\delta_2 = 0 + 0.035 = 0.035$

$\delta_3^1 = \delta_3^0 + \Delta\delta_3 = 0 - 0.184 = -0.184$

$$[\Delta V_3] = -[-20]^{-1}\left[\frac{-1.3}{1}\right] = -0.065$$

$\therefore V_3^1 = V_3^0 + \Delta V_3 = 1 + (-0.065) = 0.935$ pu

$\therefore$

$V_1^1 = 1.05 \angle 0^\circ$

$V_2^1 = 1.02 \angle 0.035^\circ$

$V_3^1 = 0.935 \angle -0.184^\circ$

All values in p.u.

Q) Consider the same Question, with the reactive power limit $-10 < Q_2 < 100$

Sol$^n$:

**Step 1:** Calculate Ybus:

$$\begin{bmatrix} 30\angle -1.57 & 20\angle 1.57 & 10\angle 1.57 \\ 20\angle 1.57 & 30\angle -1.57 & 10\angle 1.57 \\ 10\angle 1.57 & 10\angle 1.57 & 20\angle -1.57 \end{bmatrix}$$

**Step 2:** Set Initial values; $V_1^0 = 1.05\angle 0^0 \quad V_2^0 = 1.02\angle 0^0 \quad V_3^0 = 1\angle 0^0$.

**Step 3:** Check for Reactive power limit

Same as before $Q_2 = -0.408\ pu$

Limit $\frac{-10}{100} < Q_2 < \frac{100}{100} \quad = -0.1 < Q_2 < 1$

$\therefore$ Limit violated; $\therefore$ Set $Q_2$ = violated limit
$= -0.1\ pu$ and treat the bus as PQ bus.

$\therefore$ Set $V_2^0 = 1\angle 0^0$.

**Step 4:** Calculate $\Delta P$ & $\Delta Q$.

$\Delta P = P_{spec} - P_{cal}$. Specified values remains same as in previous case.

$P_{2\,cal} = 0.0486$ ; $P_{3\,cal} = 0.0323$ $Q_{3\,cal} = 0.499$ ; $Q_{2\,cal} = -0.1$

$Q_{2\,spec} = 0$

$\Delta P_2 = 2.9514$

$\Delta P_3 = -4.0323$

$\Delta Q_2 = 0.1$

$\Delta Q_3 = -1.5$

Step 5: Bus Susceptance matrix

$$B' = \begin{bmatrix} -30 & 10 \\ 10 & -20 \end{bmatrix} = B''$$

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \end{bmatrix} = -\begin{bmatrix} B_{22} & B_{23} \\ B_{32} & B_{33} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\Delta P_2}{|V_2|} \\ \frac{\Delta P_3}{|V_3|} \end{bmatrix}$$

$$\begin{bmatrix} \Delta V_2 \\ \Delta V_3 \end{bmatrix} = -\begin{bmatrix} B_{22} & B_{23} \\ B_{32} & B_{33} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\Delta Q_2}{V_2} \\ \frac{\Delta Q_3}{|V_3|} \end{bmatrix}$$

Step 6:

$$\therefore \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \end{bmatrix} = -\begin{bmatrix} -30 & 10 \\ 10 & -20 \end{bmatrix}^{-1} \begin{bmatrix} \frac{2.9514}{1} \\ \frac{-4.0323}{1} \end{bmatrix} = \begin{bmatrix} 0.0374 \\ -0.1829 \end{bmatrix}$$

$$\therefore \delta_2' = 2.14^0$$

$$\delta_3' = -10.48^0$$

lly

$$\begin{bmatrix} \Delta V_2 \\ \Delta V_3 \end{bmatrix} = -\begin{bmatrix} -30 & 10 \\ 10 & -20 \end{bmatrix}^{-1} \begin{bmatrix} \frac{0.1}{1} \\ \frac{-1.5}{1} \end{bmatrix} = \begin{bmatrix} 0.026 \\ 0.088 \end{bmatrix}$$

$$\therefore V_2' = 1.026$$

$$V_3' = 1.088$$

$$\therefore$$

$$V_1' = 1.05\angle 0^0$$

$$V_2' = 1.026\angle 2.14^0$$

$$V_3' = 1.088\angle -10.48^0$$

# MODULE - 4.

This chapter deals with the control of active and reactive power in order to keep the system in the steady-state. The objective of the control strategy is to generate and deliver power in an interconnected system as economically and reliably as possible while maintaining the voltage and frequency within permissible limits.

Changes in real power affect mainly the system frequency, while reactive power is less sensitive to changes in frequency and is mainly dependent on changes in voltage magnitude. Thus, real and reactive powers are controlled seperately.

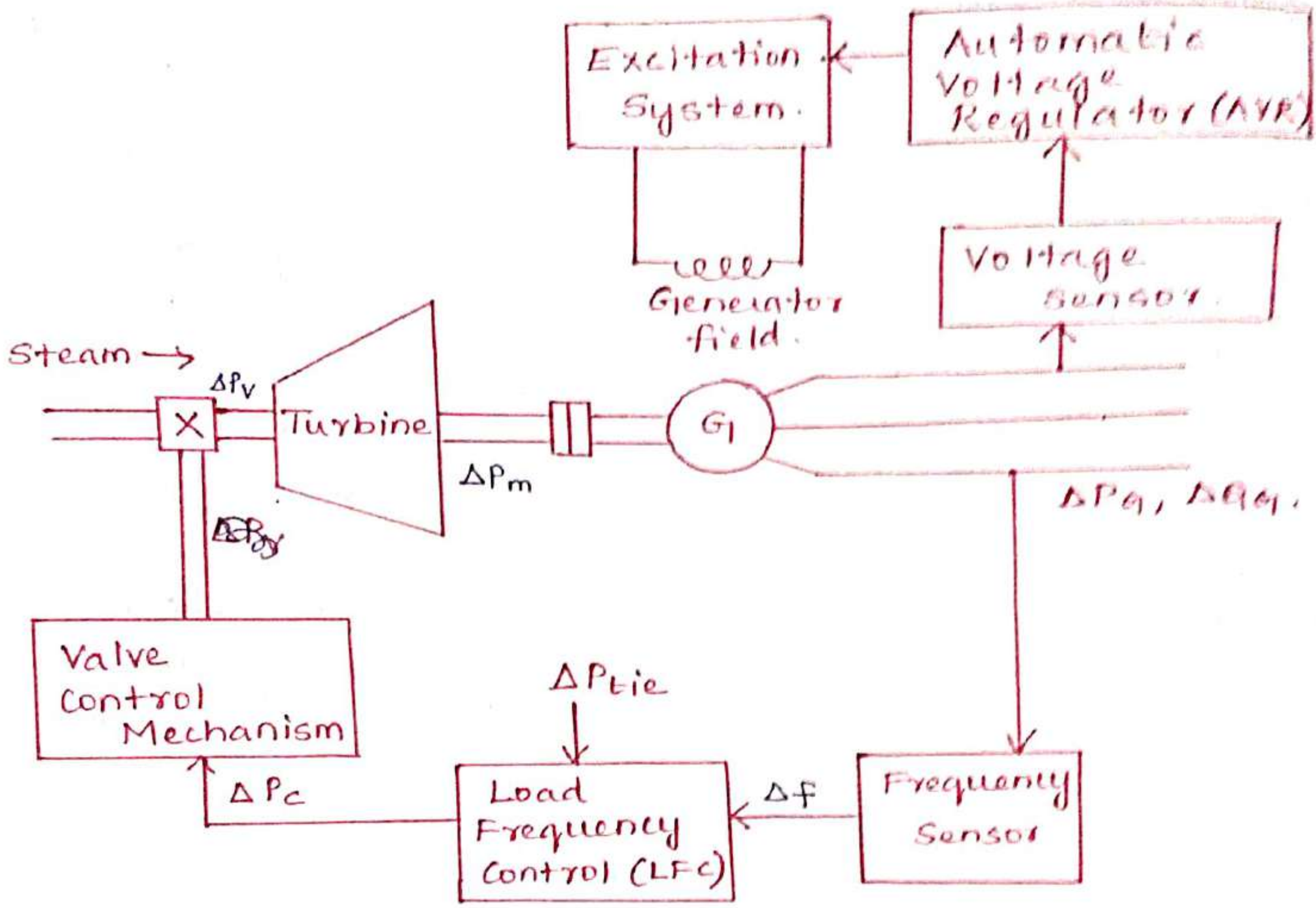
The load frequency control (LFC) loop controls the real power and frequency and the automatic voltage regulator (AVR) loop regulates the reactive power and voltage magnitude.

## BASIC GENERATOR CONTROL LOOPS :-

In an interconnected power system, LFC and AVR equipment are installed for each generator. The controllers are set for a particular operating condition and take care of small changes in load demand to maintain the frequency and voltage magnitude within the specified limits.

Small changes in real power are mainly dependent on changes in rotor angle $\delta$ and thus, the frequency.

The reactive power is mainly dependent on the voltage magnitude (ie on the generator excitation).




## LOAD FREQUENCY CONTROL (SINGLE AREA).

The objectives of the LFC are to maintain reasonably uniform frequency, to divide the load between generators, and to control the tie-line interchange schedules.

The change in frequency and tie-line real power are sensed, which is a measure of the change in rotor angle $\delta$, ie the error $(\Delta\delta)$ to be corrected. The error signal, ie $\Delta f$ and $\Delta P_{tie}$, are amplified, mixed and transformed into a real power command signal $\Delta P_V$, which is sent to the prime mover to call for an increment in torque.

The prime mover, therefore, brings change in the generator output by an amount $\Delta P_g$, which will change the values of $\Delta f$ and $\Delta P_{tie}$ within the specified tolerance.

The first step in the analysis and design of a control system is mathematical modeling of the system.

## GENERATOR MODEL :-

The swing eqn. of a synchronous machine,

$$\frac{2H}{\omega_s}\frac{d^2\Delta\delta}{dt^2} = \Delta P_m - \Delta P_e$$

velocity = distance/time, angular speed = velocity/time

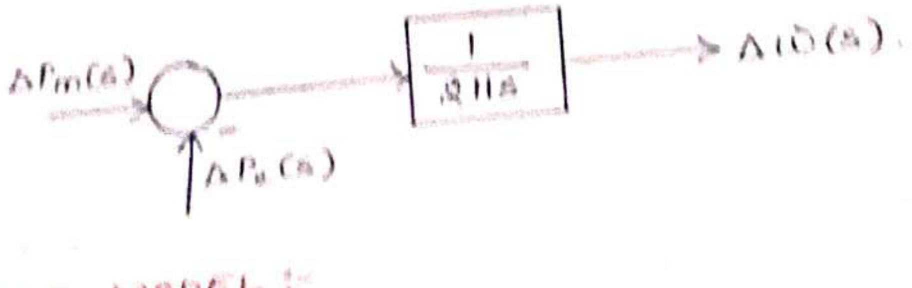
in terms of small deviation in speed, we

$$\frac{d\Delta\frac{\omega}{\omega_s}}{dt} = \frac{1}{2H}(\Delta P_m - \Delta P_e).$$

$$\frac{d\Delta\omega}{dt} = \frac{1}{2H}(\Delta P_m - \Delta P_e).$$

$$s\Delta\omega(s) = \frac{1}{2H}(\Delta P_m(s) - \Delta P_e(s)).$$

$$\Delta\omega(s) = \frac{1}{2Hs}(\Delta P_m(s) - \Delta P_e(s)).$$




## LOAD MODEL :-

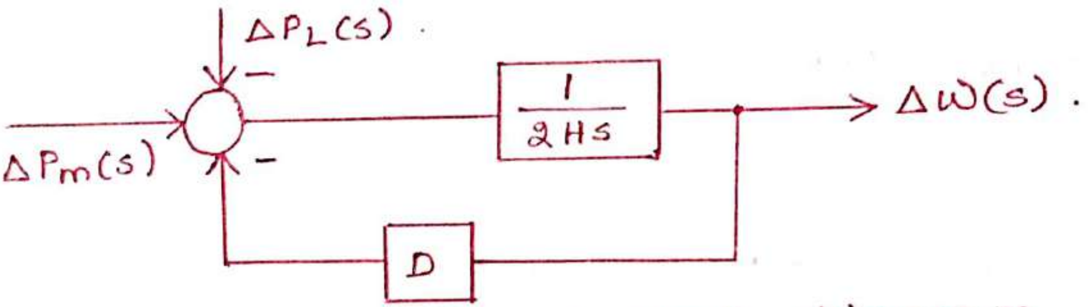
The load on a power system consists of variety of electrical devices. For resistive loads, the electrical power is independent of frequency. Motor loads are sensitive to changes in frequency

$$\Delta P_e = \Delta P_L + D\Delta\omega .$$

$\Delta P_L \rightarrow$ non frequency sensitive load change.

$D\Delta\omega \rightarrow$ frequency sensitive load change.

$D$ is the percent change in load divided by percent change in frequency.



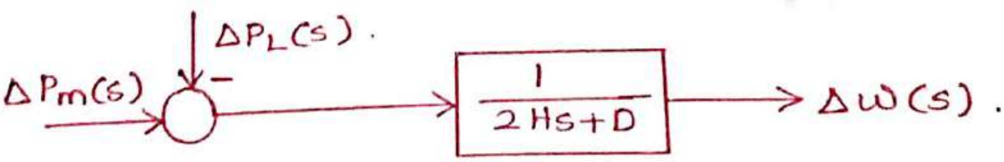

Generator and load block diagram.

$$[\Delta P_m(s) - \Delta P_e(s)] = 2Hs\ \Delta\omega(s).$$

$$\frac{\Delta P_m(s) - \Delta P_L(s) - D\Delta\omega(s)}{2Hs} = \Delta\omega(s).$$

$$\Delta P_m(s) - \Delta P_L(s) = \Delta\omega(s)\,2Hs + D\Delta\omega(s).$$

$$\frac{\Delta P_m(s) - \Delta P_L(s)}{2Hs + D} = \Delta\omega(s).$$




## TURBINE MODEL :-

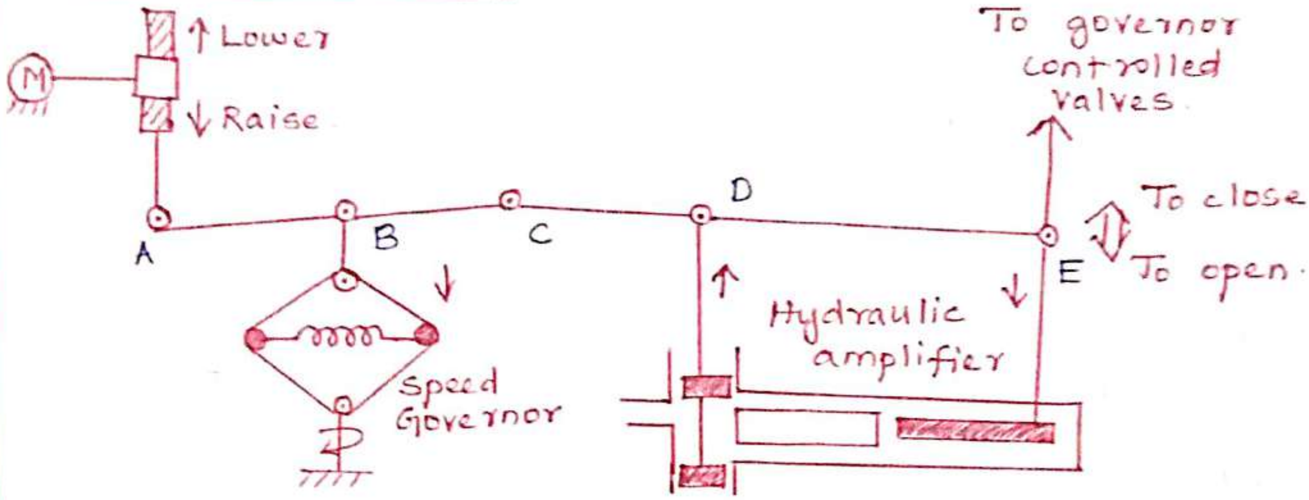
The model for the turbine relates changes in mechanical power output $\Delta P_m$ to changes in steam valve position $\Delta P_v$.

$$\frac{\Delta P_m(s)}{\Delta P_v(s)} = \frac{1}{1+\tau_T s}.$$

$$\Delta P_V(s) \longrightarrow \boxed{\frac{1}{1+\tau_T s}} \longrightarrow \Delta P_m(s).$$

The time constant $\tau_T$ is in the range of 0.2 to 2 seconds.

## GOVERNOR MODEL :-




It consists of the following parts:

### 1) Speed Governor :-

* Heart of the system which senses the change in speed (frequency).

* As the speed increases, the flyballs move outwards and the point B on linkage mechanism moves downwards. The reverse happens when the speed decreases.

### 2) Hydraulic Amplifier :-

* It comprises a pilot valve and main piston arrangement.

* Low power level pilot valve movement is converted into high power level piston valve movement. This is necessary in order to open or close the steam valve against high pressure steam.

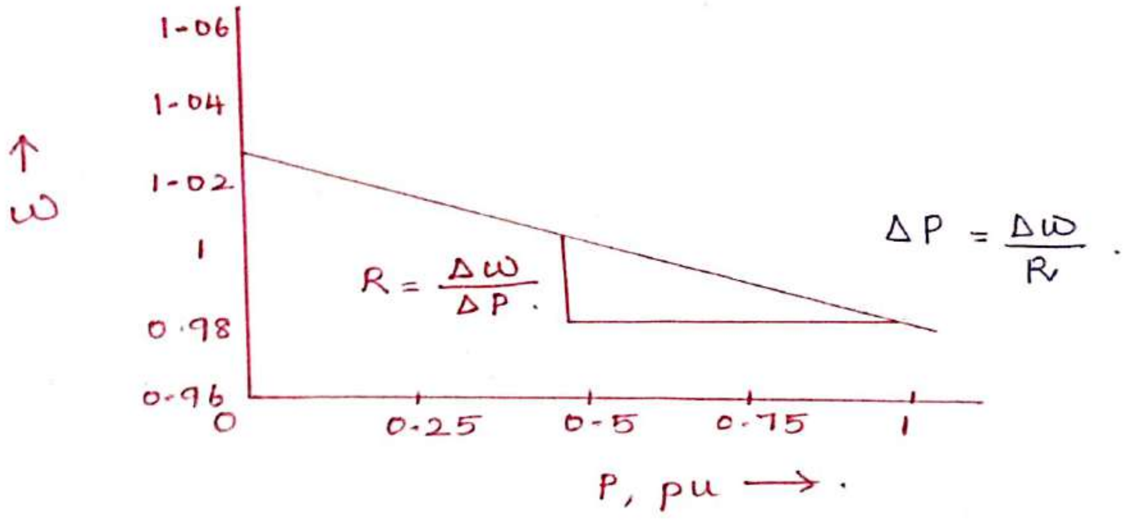
3) Linkage Mechanism :-

These are links for transforming the flyballs movement to the turbine valve through a hydraulic amplifier and providing a feedback from the turbine valve movement.

4) Speed Changer :-

It consists of a servomotor which can be operated manually or automatically for scheduling load at nominal frequency.

For stable operation, the governors are designed to permit the speed to drop as the load is increased. The steady state characteristics of a governor is shown.




The slope of the curve represents the speed regulation R.

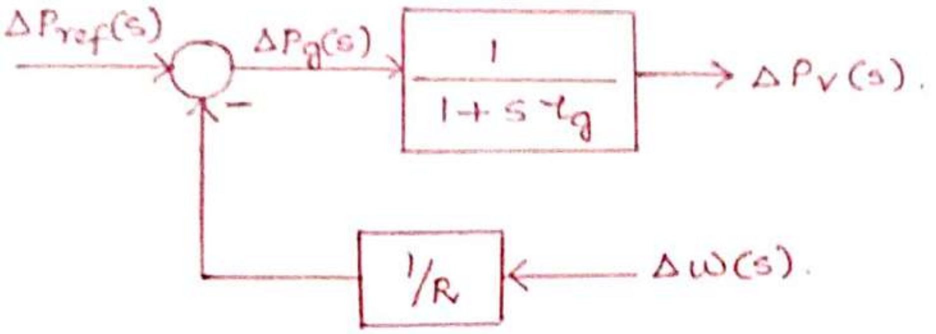
The speed governor mechanism acts as a comparator whose output $\Delta P_g$ is given as,

$$\Delta P_g = \Delta P_{ref} - \frac{1}{R} \Delta \omega.$$

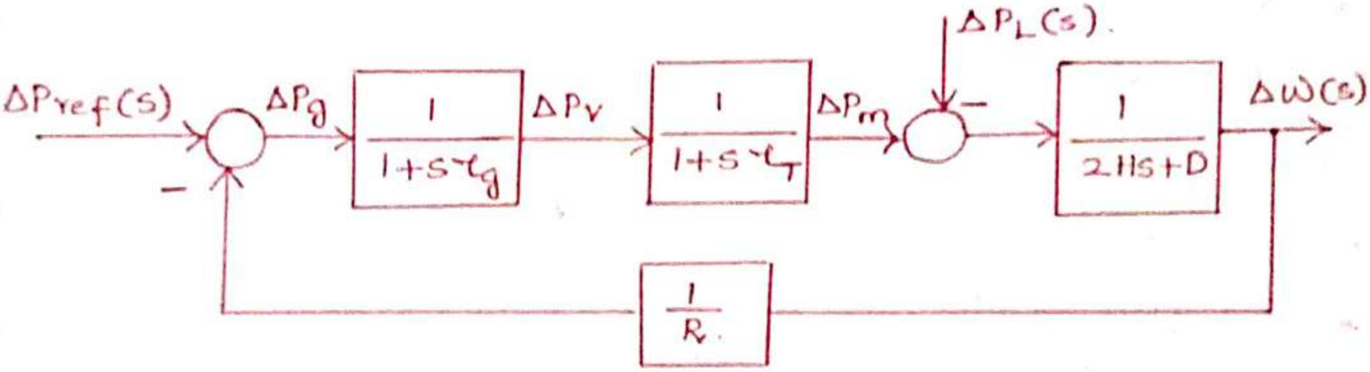
$$\Delta P_g(s) = \Delta P_{ref}(s) - \frac{1}{R} \Delta \omega(s).$$

The command $\Delta P_g$ is transformed through the hydraulic amplifier to the steam valve position command $\Delta P_V$. Hence, assuming a linear relationship, we can write, ①

$$\Delta P_V(s) = \frac{1}{1+\tau_g s} \Delta P_g(s).$$




Hence, the complete block diagram is given as,




The steady state value of $\Delta\omega$ is,

$$\Delta\omega_{ss} = (-\Delta P_L)\frac{1}{D+\frac{1}{R}} \quad , \quad \Delta\omega_{ss} = \frac{\Delta f}{f^0}$$

In case with no frequency sensitive load,

ie $D=0$, $\Delta\omega_{ss} = (-\Delta P_L)R$.

With several generators and with governor speed regulations $R_1, R_2, \ldots, R_n$,

$$\Delta\omega_{ss} = (-\Delta P_L)\frac{1}{D+\frac{1}{R_1}+\frac{1}{R_2}+\cdots+\frac{1}{R_n}}$$

## AUTOMATIC GENERATION CONTROL :-

- If the load on the system is increased, the turbine speed drops before the governor can adjust the input of the steam to the new load.

- As the change in the value of speed diminishes, the error signal becomes smaller and the position of the governor flyballs gets closer to the point required to maintain a constant speed

- However, the constant speed will not be the set point, and there will be an offset.

- One way to restore the speed or frequency to its nominal value is to add an integrator. The integral unit monitors the average error over a period of time and will overcome the offset.

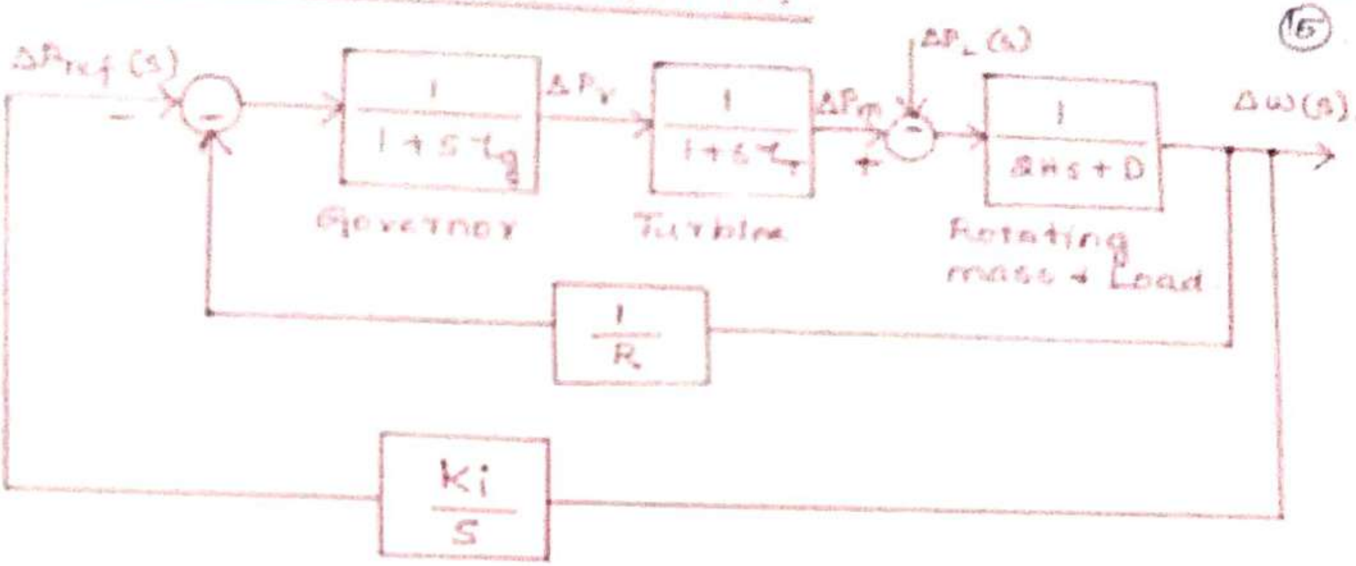
- Because of its ability to return a system to its set point, integral action is also known as reset action.

- Thus as system load changes continuously, the generation is adjusted automatically to restore the frequency to the nominal value. This scheme is known as Automatic Generation Control (AGC).

- In an interconnected system consisting of several pools, the role of the AGC is to divide the loads among system, stations and generators so as to achieve maximum economy and correctly control the scheduled interchanges of tie-line power while maintaining a reasonably uniform frequency.

IN A SINGLE AREA SYSTEM:-






LFC IN THE MULTIAREA SYSTEM:-

The AGC of a multiarea system can be realized by studying first the AGC for a two-area system.

Consider two areas represented by an equivalent generating unit interconnected by a lossless tie line with reactance $x_{tie}$

Each area is represented by a voltage source behind an equivalent reactance.

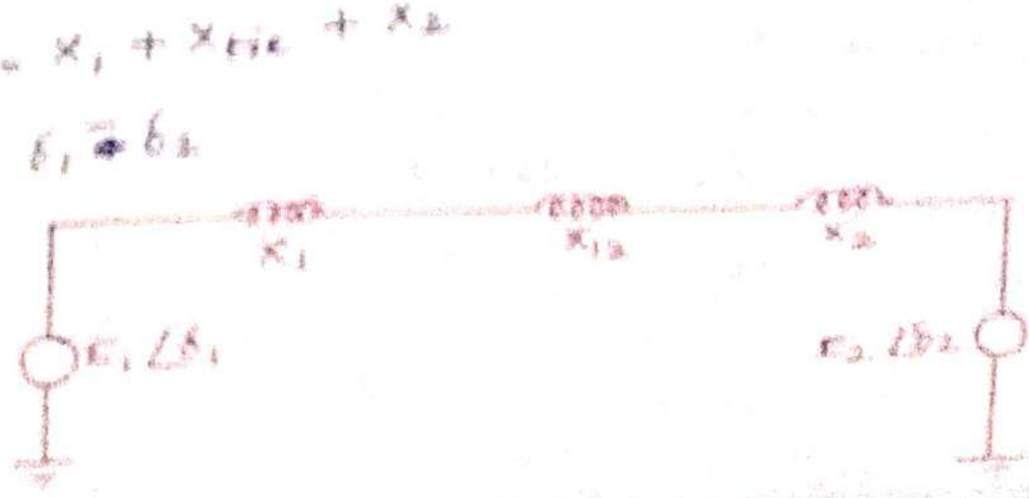
During normal operation, the real power transferred over the tie line is given by,

$$P_{12} = \frac{|E_1||E_2|}{X_{12}} \sin \delta_{12}$$

where,

$X_{12} = X_1 + X_{tie} + X_2$

$\delta_{12} = \delta_1 - \delta_2$



For a small deviation in the tie-line flow, $\Delta P_{12}$ from the nominal value,

$$\Delta P_{12} = \left.\frac{\partial P_{12}}{\partial \delta_{12}}\right|_{\delta_{12_0}} \Delta \delta_{12}.$$

$$\delta_{12_0} = \delta_{1_0} - \delta_{2_0}.$$

$P_S \rightarrow$ synchronising power coefficient.

$P_S$ = slope of the power angle curve at the initial operating angle.

$$\delta_{12_0} = \delta_{1_0} - \delta_{2_0}.$$

$$P_S = \left.\frac{\partial P_{12}}{\partial \delta_{12}}\right|_{\delta_{12_0}} = \frac{|E_1||E_2|}{X_{12}} \cos \Delta \delta_{12_0}.$$

Hence,

$$\Delta P_{12} = P_S \cdot \Delta \delta_{12}$$

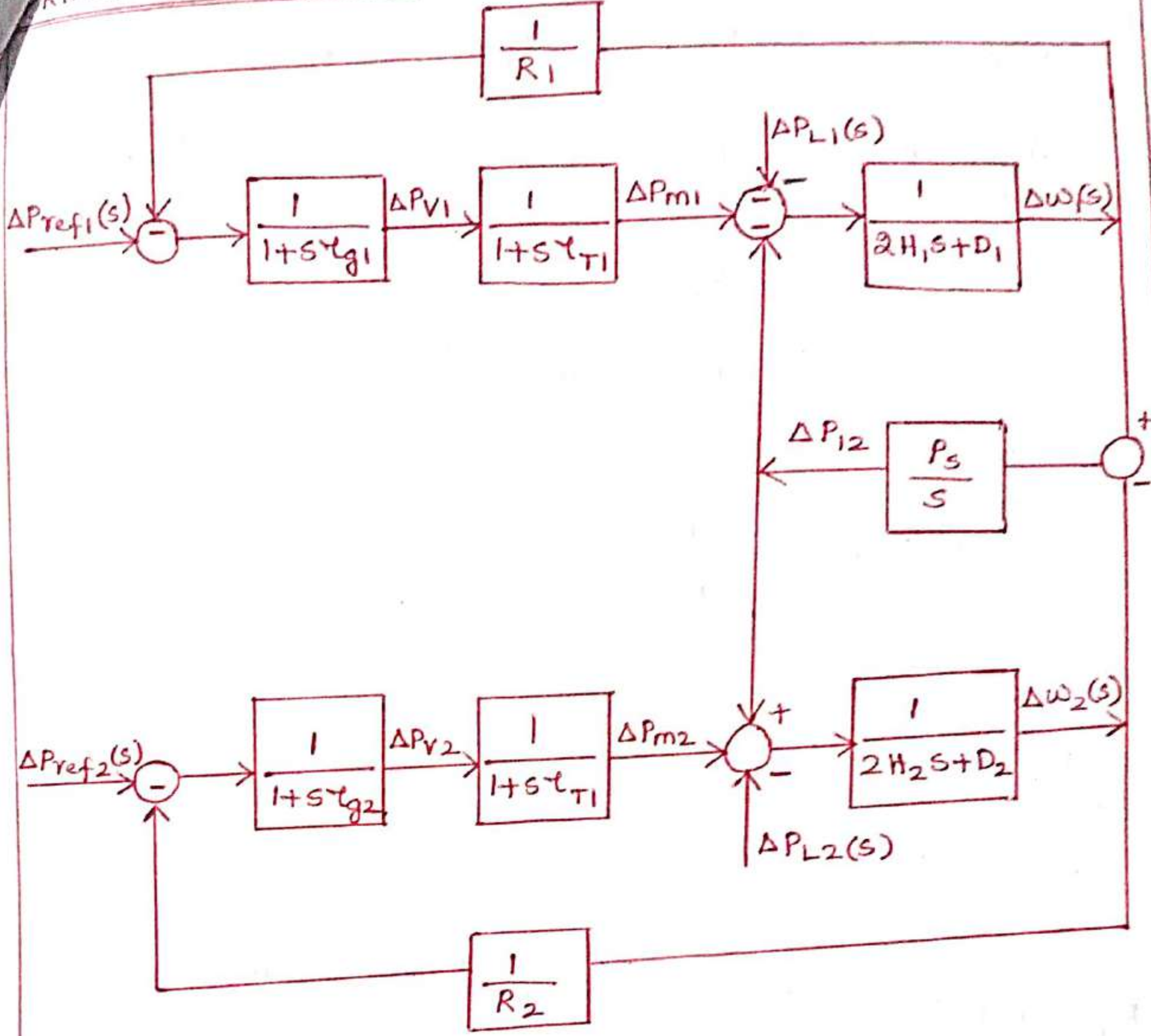
$$\Delta P_{12} = P_S (\Delta \delta_1 - \Delta \delta_2).$$

The tie line power flow appears as a load increase in one area and a load decrease in the other area, depending on the direction of the flow.

The direction of flow is dictated by the phase angle difference.

ie if $\Delta \delta_1 > \Delta \delta_2$, the power flows from area 1 to area 2.

## BLOCK DIAGRAM OF TWO AREA SYSTEM WITH ONLY PRIMARY LFC LOOP :-




Let us consider a load change $\Delta P_{L1}$ in area 1. In the steady state, both areas will have the same steady state frequency deviation, ie

$$\Delta\omega = \Delta\omega_1 = \Delta\omega_2 .$$

and

$$\Delta P_{m1} - \Delta P_{12} - \Delta P_{L1} = \Delta\omega D_1 .$$

$$\Delta P_{m2} + \Delta P_{12} = \Delta\omega D_2 \quad [\text{since } \Delta P_{L2} = 0].$$

The change in mechanical power is determined by the governor speed characteristics, given by,

$$\Delta P_{m1} = \frac{-\Delta\omega}{R_1} \quad ; \quad \Delta P_{m2} = \frac{-\Delta\omega}{R_2} .$$

$$-\Delta P_{12} = \Delta P_{m2} - \Delta\omega D_2 .$$

$$\Delta P_{m1} + \Delta P_{m2} - \Delta\omega D_2 - \Delta P_{L1} = \Delta\omega D_1 .$$

$$+\frac{\Delta\omega}{R_1} + \frac{\Delta\omega}{R_2} + \Delta\omega D_2 + \Delta\omega D_1 = -\Delta P_{L1} .$$

$$\Delta\omega = \frac{-\Delta P_{L1}}{\left(\frac{1}{R_1} + D_1\right) + \left(\frac{1}{R_2} + D_2\right)} .$$

$$\boxed{\Delta\omega = \frac{-\Delta P_{L1}}{B_1 + B_2}}$$

where,

$$B_1 = \frac{1}{R_1} + D_1$$

$$B_2 = \frac{1}{R_2} + D_2$$

$B_1$ and $B_2$ are frequency bias factors.

The change in tie-line power is,

$$\Delta P_{12} = \Delta P_{m1} - \Delta P_{L1} - \Delta\omega D_1$$

$$= \frac{-\Delta\omega}{R_1} - \Delta P_{L1} - \Delta\omega D_1 .$$

$$\Delta P_{12} = -\Delta\omega\left(\frac{1}{R_1} + D_1\right) - \Delta P_{L1}$$

$$= -\Delta\omega B_1 - \Delta P_{L1} .$$

$$= \frac{\Delta P_{L1}}{B_1 + B_2} \cdot B_1 - \Delta P_{L1} .$$

$$\boxed{\Delta P_{12} = \frac{B_2}{B_1 + B_2}\left(-\Delta P_{L1}\right)}$$

## AGC (2 AREA SYSTEM) :-

When LFC's were equipped with only the primary control loop, a change of power in area 1 was met by the increase in generation in both areas associated with a change in the tie-line power, and a reduction in frequency.

In the normal operating state, the power system is operated so that the demands of areas are satisfied at the nominal frequency.

A simple control strategy for the normal mode is,

- Keep frequency approximately at the nominal value.
- Maintain the tie line flow at schedule.
- Each area should absorb its own load changes.

...nventional LFC is based upon tie-line bias ...ntrol, where each area tends to reduce the area control error (ACE) to zero.

The control error for each area consists of a linear combination of frequency and tie-line error.

$$ACE_i = \sum_{j=1}^{n} \Delta P_{ij} + k_j \Delta \omega.$$

The area bias $k_j$ determines the amount of interaction during a disturbance in the neighbouring areas.

An overall satisfactory performance is achieved when $k_j$ is selected equal to the frequency bias factor of that area. ie $B_j = \frac{1}{R_j} + D_j$.

Thus the ACEs for a two area system are,

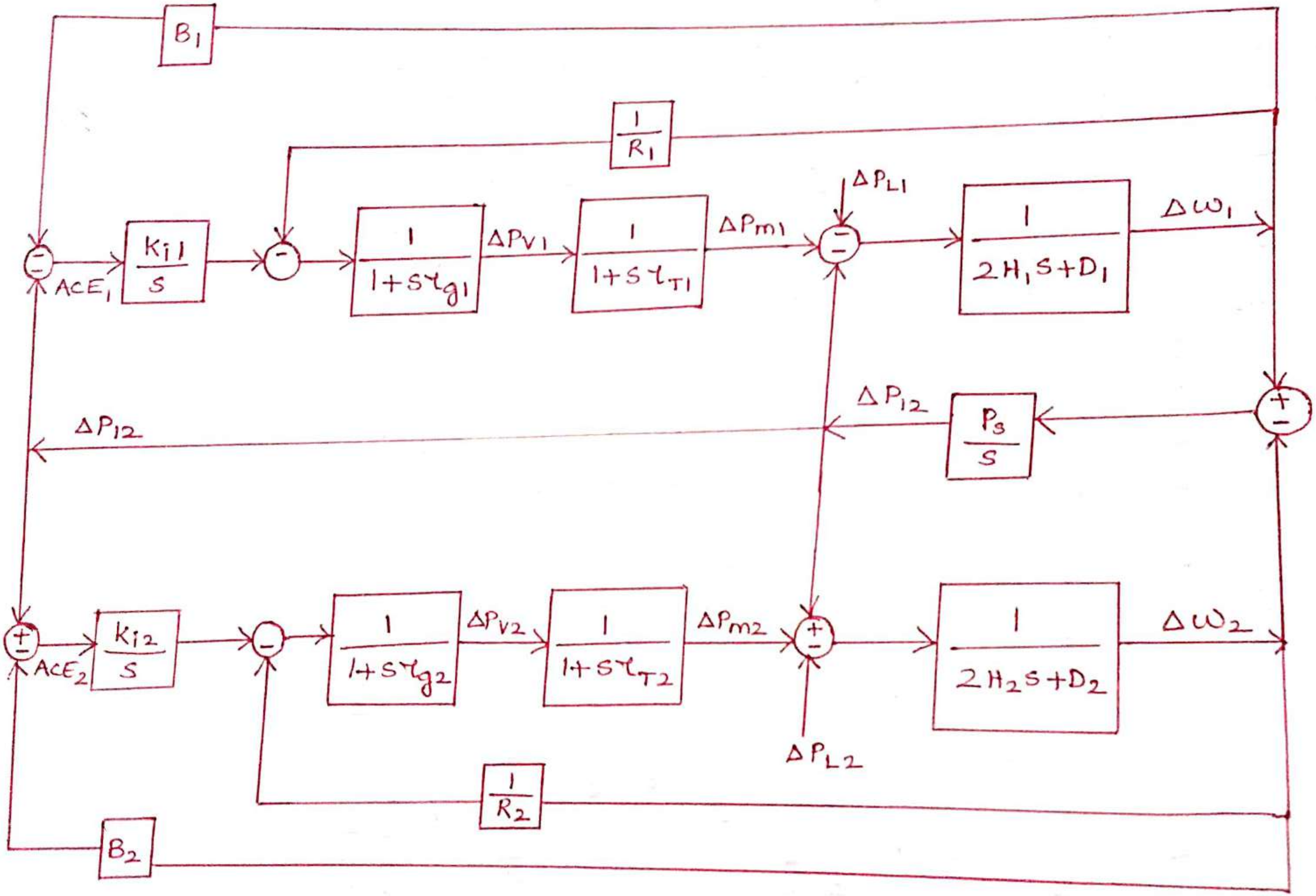
$$ACE_1 = \Delta P_{12} + B_1 \Delta \omega_1.$$

$$ACE_2 = \Delta P_{21} + B_2 \Delta \omega_2.$$

$\Delta P_{12}, \Delta P_{21} \rightarrow$ deviations from scheduled interchanges.

ACEs are used as actuating signals to activate changes in the reference power set points and when steady state is reached, $\Delta P_{12}$ and $\Delta \omega$ will be zero,

## AGC BLOCK DIAGRAM FOR A TWO-AREA SYSTEM:-



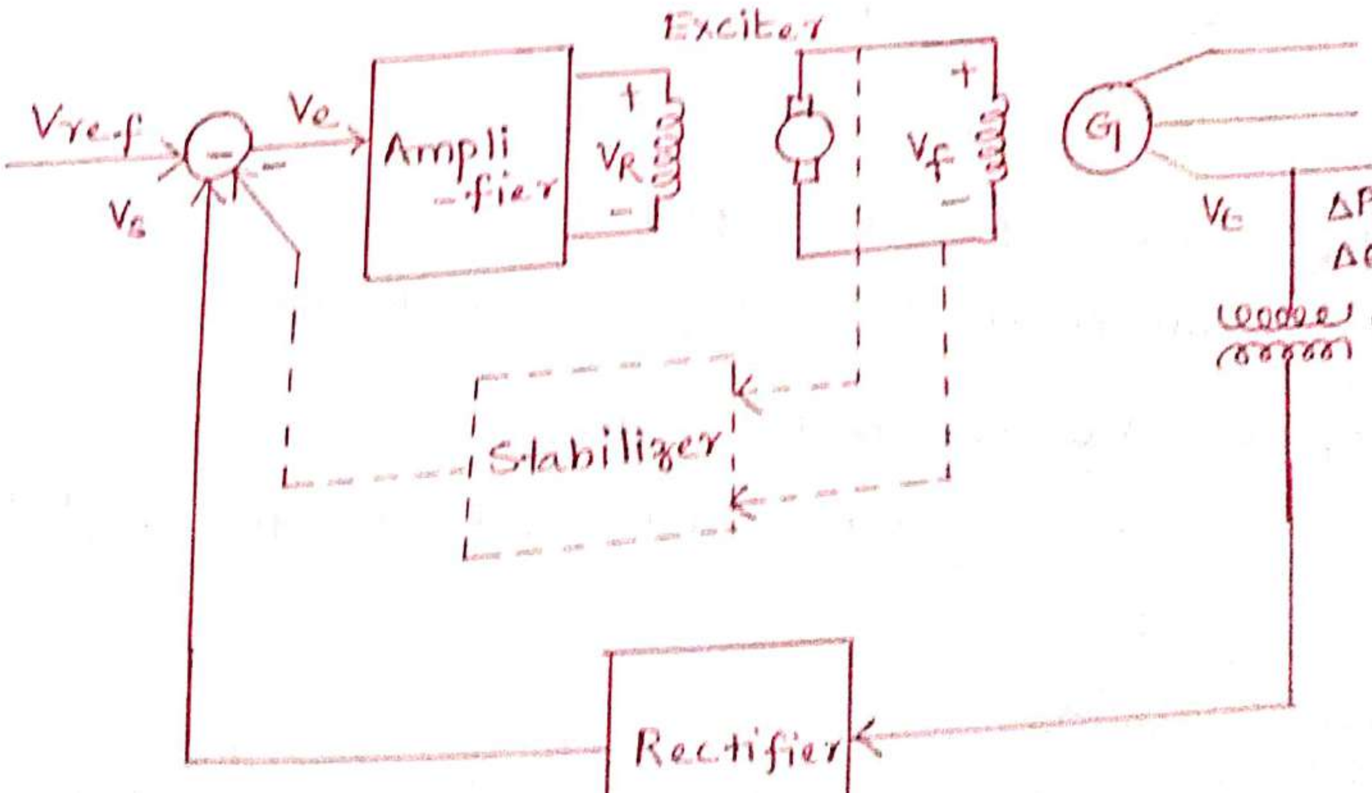
## ...OMATIC VOLTAGE CONTROL :

- A change in the real power demand affects essentially the frequency, whereas a change in the reactive power affects mainly the voltage magnitude. The interaction between voltage and frequency controls is generally weak.

- The sources of reactive power are generators, capacitors and reactors. The generator reactive powers are controlled by field excitation. Other supplementary methods of improving the voltage profile on electric transmission systems are transformer load-tap changers, switched capacitors, etc.

- The primary means of generator reactive power control is the generator excitation control using automatic voltage regulator (AVR).

- The role of AVR is to hold the terminal voltage magnitude of a synchronous generator at a specified level.



An increase in the reactive power ... generator is accompanied by a drop in the terminal voltage magnitude.

The voltage magnitude is sensed through a potential transformer on one phase.

This voltage is rectified and compared to a dc set point signal.

The amplified error signal controls the exciter field and increases the exciter terminal voltage

Thus, the generator field current is increased, which results in an increase in the generated emf.

The reactive power generation is increased to a new equilibrium, raising the terminal voltage to the desired value.

## MODELLING OF AVR :-

### AMPLIFIER MODEL :-

$$\frac{V_R(s)}{V_e(s)} = \frac{K_A}{1+\tau_A s}$$

$K_A \rightarrow$ gain of the amplifier.
values in the range of 10 to 400.

$\tau_A \rightarrow$ time constant.
0.02 to 0.1 second. and is often neglected.

MODE
non
...

## MODEL OF EXCITER :-

The output voltage of the exciter is a non-linear function of the field voltage because of the saturation effects in the magnetic circuit.

A model of the exciter is a linearized model, which takes into account the major time constant and ignores the saturation or other non linearities.

$$\frac{V_F(s)}{V_R(s)} = \frac{K_E}{1+\tau_E s}$$

$T_E$ is very small.

## GENERATOR MODEL :-

$$\frac{V_T(s)}{V_F(s)} = \frac{K_G}{1+\tau_G s}$$

These constants are load dependent.

$K_G \rightarrow 0.7$ to 1.

$\tau_G \rightarrow 1$ to 2 seconds from full load to no load.

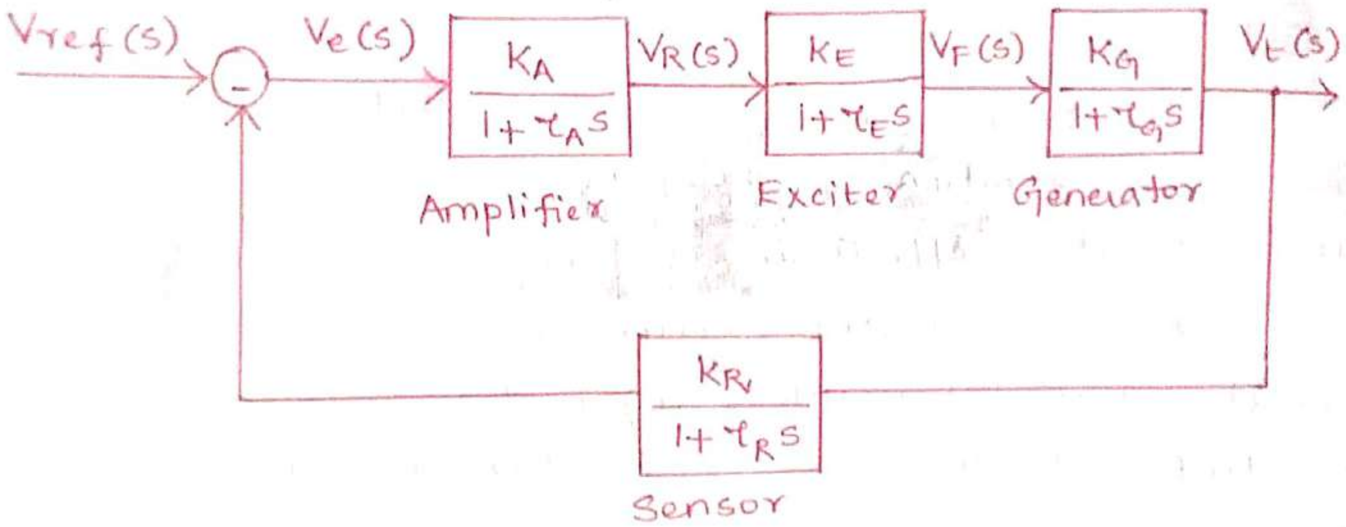
## SENSOR MODEL :-

The voltage is sensed through a potential transformer and then it is rectified through a bridge rectifier.

$$\frac{V_S(s)}{V_t(s)} = \frac{K_R}{1+\tau_R s}$$

$\tau_R$ is very small, 0.01 to 0.06 sec.

## BLOCK DIAGRAM OF AVR :-



Module IV

Reference Text: C.L. Wadhwa

# Economic Load Dispatch.

The Economic load dispatch involves the solution of two different problems. The first of these is the unit commitment or pre-dispatch problem, where in it is required to select optimally out of the available generating sources to operate, to meet the expected load and provide a specified margin of operating reserve over a specified period of time. The second aspect of economic dispatch is the on-line economic dispatch where in it is required to distribute the load among the generating units actually paralleled with the system, in such a manner as to minimize the total cost of supplying the minute to minute requirements of the system.

In case of economic dispatch, the generations are not fixed, but they are allowed to take values again within certain limits, so as to meet a particular load demand with minimal fuel consumption. Economic load dispatch is a solution of a large number of load flow problems and choosing the one which is optimal in the sense that it needs minimum cost of generation. Since the total cost of generation is a function of individual generation of the sources which can take values within certain constraints, the cost of generation will depend upon the system constraint for a particular load demand.

## System Constraints.

There are two types of constraints :- (i) Equality constriants and (ii). Inequality constraints. Inequality constraints are of two types : (i) Hard type (ii) Soft type. The hard type are those which are definite and specific like the tapping range of an on-load tap changing transformer. Soft type are those which have some flexibility associated with them like the nodal voltages and phase angles between the nodal voltages etc. Soft inequality constraints have been very efficiently handled by penalty function methods.

## Equality Constraints :

The equality constraints are the basic load flow equations, given by.

$P_i =$

$Q_i =$

$i = 1, 2, \ldots\ldots n$

## Inequality Constraints.

(a) Generator Constraints : The kVA loading on a generator is given by $S_i = \sqrt{P_i^2 + Q_i^2}$ and this should not exceed a pre specified value $C_i$, because of the temperature rise condition.

$$\therefore \quad P_i^2 + Q_i^2 \leq C_i^2$$

The maximum active power generation of a source is again limited by thermal consideration and also minimum power generation limited by flame instability of boiler.

$$P_i \min \leq P_i \leq P_i \max.$$

Similarly max and min reactive power generation of the source is limited

$$\therefore Q_i \min \leq Q_i \leq Q_i \max.$$

(b) Voltage Constraints: It is essential that the voltage magnitudes and phase angles at various nodes should vary within certain limits. The voltage magnitudes should vary within certain limits, because otherwise most of the equipments connected to the system will not operate satisfactorily or additional use of voltage regulating devices will make the system uneconomical, Thus

$$|v_i \min| \leq |v_i| \leq |v_i \max|$$

$$\delta_i \min \leq \delta_i \leq \delta_i \max.$$

(c) Running Spare Capacity Constraints: These constraints are required to meet (i) forced outages of one or more alternators (ii) Unexpected load on the system

$\therefore P_G \geq P_i + P_{so}$ ; where $P_{so}$ spare capacity min.

(d) Transformer tap settings: If an auto transformer is used, the min tap setting is zero and maximum one

$$\therefore 0 \leq t \leq 1$$

Similarly for two winding transformer; $0 \le t \le n$ ; $n \rightarrow$ ratio of transformation

Phase shift limits of phase shifting transformer $\theta_{i\,min} \le \theta_i \le \theta_{i\,max}$.

(e) Transmission line Constraints: The flow of active and reactive power through the transmission line circuit is limited by thermal capacity of the circuit.

$$C_i \le C_{i\,max}$$

$C_{i\,max} \rightarrow$ max loading on $i^{th}$ line.

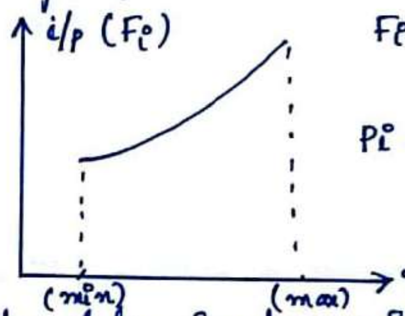
(f) Network Security Constraints:

If initially a system is operating satisfactorily and there is an outage, may be scheduled or forced one, it is natural that some of the constraints of the system will be violated.

Characteristics of Thermal Power Plant:

(i). Cost Curve: The curve drawn between input of the plant on Y axis (in Rs/hr or Kcal/hr) and net output power on X axis (MW).

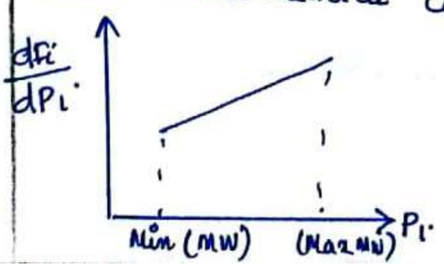
By fitting a suitable 2° polynomial the expression for the operating cost can be written as $F_i = \frac{1}{2} a_i P_i^2 + b_i P_i + c_i$




$F_i \rightarrow$ fuel input
$P_i \rightarrow$ Power o/p

where $a_i, b_i, c_i$ are constants and can be determined experimentally

(ii) Incremental cost curve: The slope of the cost curve $dF_i/dP_i$ is called incremental cost function or incremental production cost.

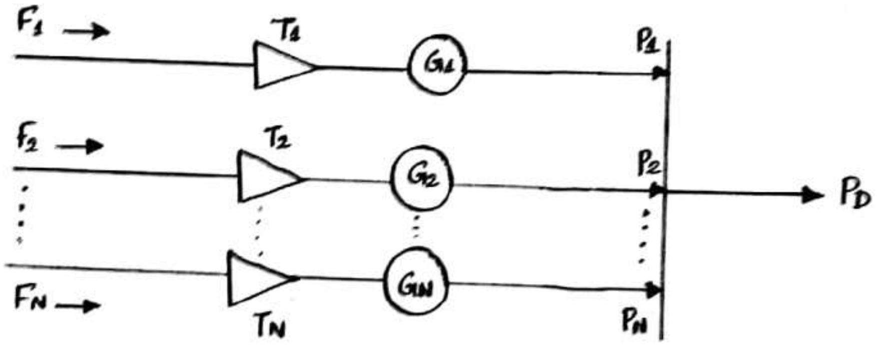



Represented in Rs/MWhr. ($I_c$)

$\therefore$ Incremental Cost $= \frac{dF_i}{dP_i}$
$= a_i P_i + b_i$

## Economic load dispatch neglecting transmission losses.

Consider, there are 'N' generation units. $F_1, F_2 \cdots F_N$ be the fuel inputs to the plants (fuel cost). $P_1, P_2 \cdots P_N$ be the power output from plants $1, 2 \ldots N$. Let $T \rightarrow$ Thermal plant, G - generator and $P_D$ be the power demand.




Our objective is to minimize the cost of production of power subjected to some constrain

$$\text{Total fuel (fuel cost)} \rightarrow F_T = F_1 + F_2 + \cdots + F_N$$
$$= \sum_{i=1}^{N} F_i$$

$$\therefore \quad \text{Objective Function} = F_T = \sum_{i=1}^{N} F_i .$$

Constrain $\rightarrow$ Since losses are neglected, total power generated = demand

$$\text{or} \quad P_D = \sum_{i=1}^{N} P_i$$

$$P_D - \sum_{i=1}^{N} P_i = 0 \longrightarrow \text{equality constrain } (\phi).$$

Hence, our problem is to minimize the objective function subjected to the equality constrain.

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For solving the problem, define a Lagrange function '$\mathcal{L}$'.

This function is obtained by adding the O.F to the constrain, after multiplying the constrain by an undetermined multiplier $\lambda$ (Lagrange multiplier).

$$\therefore \quad \mathcal{L} = O.F + \lambda \cdot \phi$$

$$\therefore \quad \mathcal{L} = \sum_{i=1}^{N} F_i + \lambda \left( P_D - \sum_{i=1}^{N} P_i \right)$$

To minimize the function, differentiate $\mathcal{L}$ w.r.t. $P_i$.

$$\therefore \quad \frac{d\mathcal{L}}{dP_i} = \frac{dF_i}{dP_i} + \lambda(0-1)$$

now equate to zero

$$\therefore \quad \frac{dF_i}{dP_i} - \lambda = 0 \quad \text{or} \quad \boxed{\frac{dF_i}{dP_i} = \lambda}$$

ie $$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2} = \cdots\cdots = \frac{dF_N}{dP_N} = \lambda.$$

$\therefore$ To minimize the cost of production; $\frac{dF_i}{dP_i} = \lambda$.

where $\frac{dF_i}{dP_i}$ is called Incremental production cost.

also we have $\boxed{\frac{dF_i}{dP_i} = a_i P_i + b_i}$

where $a_i$ and $b_i$ are constants.

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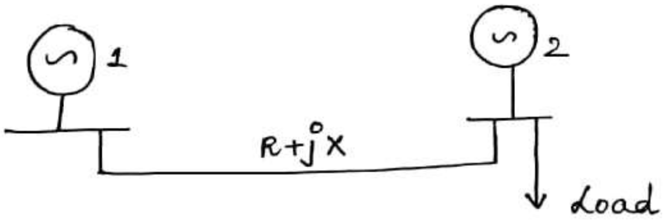
## Economic load dispatch (Optimal load dispatch) including Transmission Losses.

When the Energy is to be transported over a large distance, transmission losses in some cases may amount to 20 to 30% of total load, and it then become necessary to take the losses also into account, when developing our economic dispatch strategy.




Here, 2 generators have equal incremental production cost. If generator 2 has a local load, according to equal production cost criteria, total load must be shared by the two. But commonsense tells us that it is more economical to let Gen 2 supply most of the local load, because gen 1 has to supply in addition to the load, the transmission losses also. So here, equal incremental cost does not hold good, so we have to consider transmission losses also.

$\therefore$ Objective function $F_T = \sum_{i=1}^{N} F_i$.

constraint $\quad P_D = \sum_{i=1}^{N} P_i - P_L \quad$ ; where $P_L \rightarrow$ total transmission loss.

or $\quad P_D - \sum_{i=1}^{N} P_i + P_L = 0$

$\therefore$ Lagrange function $\mathcal{L} = O.F + \lambda\phi$

$$\therefore \quad \mathcal{L} = \sum_{i=1}^{N} F_i + \left(P_D - \sum_{i=1}^{N} P_i + P_L\right)\cdot\lambda$$

now differentiate $\mathcal{L}$ w.r.t $P_i$ and equate to zero.

$$\frac{d\mathcal{L}}{dP_i} = \frac{dF_i}{dP_i} + \lambda\left(0 - 1 + \frac{dP_L}{dP_i}\right) = 0$$

$$\therefore \quad \frac{dF_i}{dP_i} + \lambda\left(\frac{dP_L}{dP_i} - 1\right) = 0$$

$P_L$ is a function of $P_i$.

or $$\boxed{\frac{dF_i}{dP_i} + \lambda\cdot\frac{dP_L}{dP_i} = \lambda}$$ $\rightarrow$ co-ordination equation.

To solve the equation, there are 2 methods.

(1). Optimal load flow or Penalty factor method

(2) Loss formula using $\beta$- Coefficient.

## Penalty Factor Method.

$$\mathcal{L}_i = \frac{1}{1 - \frac{dP_L}{dP_i}} \simeq \left(1 + \frac{dP_L}{dP_i}\right)$$

we have $$\frac{dF_i}{dP_i} + \lambda\frac{dP_L}{dP_i} = \lambda$$

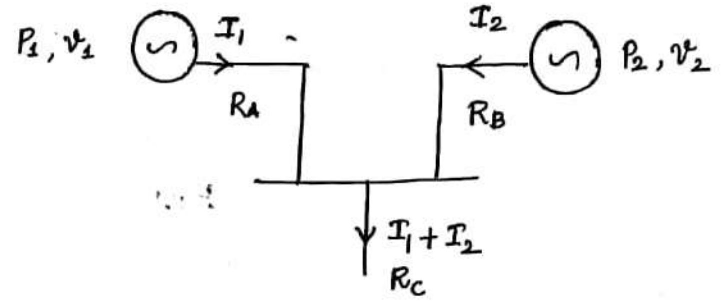
$$\frac{dF_i}{dP_i} = \lambda\left(1 - \frac{dP_L}{dP_i}\right)$$

$$\frac{dF_i}{dP_i} \times \left[\frac{1}{\left(1 - \frac{dP_L}{dP_i}\right)}\right] = \lambda \quad \text{or} \quad \boxed{\frac{dF_i}{dP_i}\cdot \mathcal{L}_i = \lambda}$$

where

Loss formula using B coefficients.

Consider the single line diagram of a two generating s/m.




Let $P_1, V_1, I_1, R_A, P_2, V_2, I_2, R_B, R_C$, be the powers, voltage, current and Resistances of generator 1 and 2 respectively. From the single line diagram, the total transmission loss: $P_L$ is:

$$P_L = 3\left\{ I_1^2 R_A + I_2^2 R_B + (I_1 + I_2)^2 R_C \right\} \longrightarrow (1)$$

Since this is 3 phase s/m;

$$P_1 = \sqrt{3}\, V_1 I_1 \cos\phi_1 \quad ; \quad P_2 = \sqrt{3}\, V_2 I_2 \cos\phi_2$$

$$\therefore I_1 = \frac{P_1}{\sqrt{3}\, V_1 \cos\phi_1} \quad ; \quad I_2 = \frac{P_2}{\sqrt{3}\, V_2 \cos\phi_2}$$

On substituting the values of $I_1$ and $I_2$ in eq$^n$ (1).

$$P_L = 3\left\{ \frac{P_1^2 \cdot R_A}{3 V_1^2 \cos^2\phi_1} + \frac{P_2^2 \cdot R_B}{3 V_2^2 \cos^2\phi_2} + \frac{P_1^2 \cdot R_C}{3 V_1^2 \cos^2\phi_1} + \frac{P_2^2 \cdot R_C}{3 V_2^2 \cos^2\phi_2} + \frac{2 P_1 P_2 R_C}{3 V_1 V_2 \cos\phi_1 \cos\phi_2} \right\}$$

$$\Rightarrow P_L = \frac{(R_A + R_C) \cdot P_1^2}{V_1^2 \cos^2\phi_1} + \frac{(R_B + R_C) \cdot P_2^2}{V_2^2 \cos^2\phi_2} + \frac{2 P_1 \cdot P_2 \cdot R_C}{V_1 \cdot V_2 \cdot \cos\phi_1 \cdot \cos\phi_2}.$$

on re arranging,

$$P_L = P_1 \cdot \underbrace{\left[\frac{R_A + R_C}{V_1^2 \cos^2\phi_1}\right]}_{B_{11}} P_1 + P_2 \underbrace{\left[\frac{R_B + R_C}{V_2^2 \cos^2\phi_2}\right]}_{B_{22}} P_2 + P_1 \underbrace{\left[\frac{R_C}{V_1 V_2 \cos\phi_1 \cos\phi_2}\right]}_{B_{12}} P_2$$

$$+ P_2 \underbrace{\left[\frac{R_C}{V_1 V_2 \cos\phi_1 \cos\phi_2}\right]}_{B_{21}} P_1 .$$

$\therefore \quad P_L = P_1 \, B_{11} \, P_1 + P_2 \, B_{22} \, P_2 + P_1 \, B_{12} \, P_2 + P_2 \, B_{21} \, P_1$

where $B_{11}$, $B_{22}$, $B_{12}$ and $B_{21}$ are $\beta$ coefficients

$\therefore$ In general; $\quad P_L = \sum_m \sum_n P_m B_{mn} P_n$ ; which is the loss formula.

$\therefore$ For a two generator s/m.

$P_L = \sum_m \sum_n P_m B_{mn} P_n$ : where m and n varies from 1 to 2.

$\therefore \quad P_L = P_1 \, B_{11} \, P_1 + P_1 \, B_{12} \cdot P_2 + P_2 \, B_{22} \, P_2 + P_2 \, B_{21} \, P_1$

to find $\frac{dP_L}{dP_i}$ ; if $i = 1$.

$$\frac{dP_L}{dP_1} = 2 P_1 B_{11} + B_{12} \cdot P_2 + B_{21} \cdot P_2 \qquad \left\{ \text{Since } B_{12} = B_{21} \right\}$$

$$= 2 P_1 B_{11} + 2 \, P_2 \cdot B_{12}$$

$$= 2 \left( P_1 B_{11} + P_2 B_{12} \right)$$

$\therefore$ In general,

$$\frac{dP_L}{dP_i} = 2 \sum_m B_{im} \cdot P_m .$$

we know the co-ordinate equation is $\frac{dF_i}{dP_i} + \lambda \cdot \frac{dP_L}{dP_i} = \lambda$

ie $$\frac{dF_i}{dP_i} + \lambda \left\{ 2 \sum_m B_{im} \cdot P_m \right\} = \lambda$$

Since $\frac{dF_i}{dP_i} = a_i P_i + b_i$

$$a_i P_i + b_i + \lambda \left\{ 2 \sum_m B_{im} \cdot P_m \right\} = \lambda$$

$$a_i P_i + b_i + \lambda \, 2 B_{ii} P_i + \lambda \left\{ 2 \sum_{m \neq i} B_{im} \cdot P_m \right\} = \lambda$$

$$P_i \left( a_i + 2 \lambda B_{ii} \right) = \lambda - b_i - \lambda \left\{ 2 \sum_{m \neq i} B_{im} P_m \right\}$$

$$P_i = \frac{\lambda - b_i - \lambda \left\{ 2 \sum_{m \neq i} B_{im} \cdot P_m \right\}}{a_i + 2 \lambda B_{ii}}$$

$$\boxed{P_i = \frac{1 - \frac{b_i}{\lambda} - 2 \sum_{m \neq i} B_{im} \cdot P_m}{\frac{a_i}{\lambda} + 2 B_{ii}}}$$

The formula for power loss using β coefficients is derived using the following assumptions.

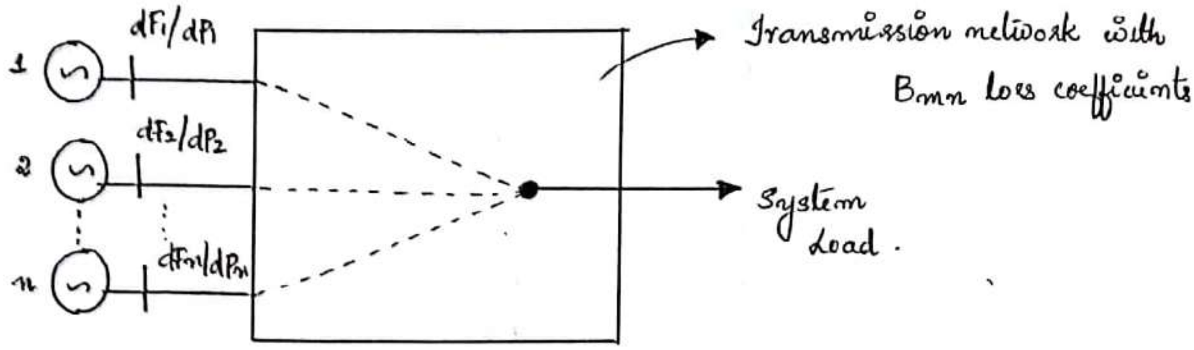
(1). The equivalent load current at any bus is a constant complex fraction of the total equivalent current.

(2). Generator bus voltage magnitude and angles are constant.

(3). Power factor of each source is constant.

## Physical Interpretation of Co-ordination Equation.

The physical interpretation of the co-ordination equation can be understood with the help of figure shown below.




Let there are 'n' number of plants connected to a hypothetical load through a transmission network. The incremental cost of production at the $n^{th}$ bus bar is $dF_n/dP_n$. Let there is an increase in power demand (load) $\Delta P_D$. Let this increase in demand is met by plant 'n' alone.

$\therefore$ Let $\Delta P_n$ be the increase in power generation at plant 'n' to meet the increase in demand $\Delta P_D$ and increase in transmission loss $\Delta P_L$.

$$\therefore \quad \Delta P_n = \Delta P_D + \Delta P_L.$$

Since the incremental cost of power at plant $n = dF_n/dP_n$ Rs/MW hr,

Cost of power at the plant bus for an additional generation of $\Delta P_n$ is

$$\frac{dF_n}{dP_n} \cdot \Delta P_n \quad \text{Rs/hr}.$$

Since the power at the receiving end is only $\Delta P_D$ (Since loss present, $\Delta P_n \neq \Delta P_D$), the cost of received power is,

$$\lambda = \frac{dF_n}{dP_n} \cdot \frac{\Delta P_n}{\Delta P_D}$$

$$= \frac{dF_n}{dP_n} \cdot \frac{\Delta P_n}{\Delta P_n - \Delta P_L} \qquad \left\{ \text{since } \Delta P_n = \Delta P_D + \Delta P_L \right\}$$

$$= \frac{dF_n}{dP_n} \cdot \frac{1}{1 - \frac{\Delta P_L}{\Delta P_n}}$$

$$= \frac{dF_n}{dP_n} \cdot \frac{1}{1 - \frac{dP_L}{dP_n}}$$

$$\therefore \quad \lambda = \frac{dF_n}{dP_n} \cdot L_n \quad ; \text{ where } L_n = \frac{1}{1 - \frac{dP_L}{dP_n}}.$$

which is same as the co-ordination equation and from this treatment, penalty factor for plant n can be defined as the ratio of small change in power at plant 'n' to the small change in received power when generation at plant 'n' alone is changed to meet the load.

## Exact Transmission loss Formula.

Here a formula for calculating transmission losses ($P_L$) by making use of bus powers and system parameters.

Let $S_i$ be the total power at bus 'i', which is equal to generated power at bus 'i' minus load power at bus 'i'. This means the net power at bus 'i' corresponds to losses. The summation of all such powers at all buses gives the total losses in the system.

ie,

$$P_L + jQ_L = \sum_{i=1}^{n} S_i$$

$$= \sum_{i=1}^{n} V_i I_i^*$$

{ where $P_L$ = active component of loss.

$Q_L$ = Reactive comp of loss.

$S_i$ = Complex power at $i^{th}$ bus

$= V_i . I_i^*$.

$$V_{bus} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \quad I_{bus} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} \quad I_{bus}^* = \begin{bmatrix} I_1^* \\ I_2^* \\ \vdots \\ I_n^* \end{bmatrix}$$

$$\sum_{i=1}^{n} V_i I_i^* = V_1 . I_1^* + V_2 I_2^* + \ldots V_n . I_n^* .$$

$$= [V_{bus}]^T [I_{bus}^*]$$

$\therefore$

$$P_L + jQ_L = V_{bus}^T I_{bus}^*$$

$$= [Z_{bus} . I_{bus}]^T I_{bus}^*$$

{ $V = Z . I$

$\therefore V_{bus} = Z_{bus} . I_{bus}$.

$$= I_{bus}^T . Z_{bus}^T I_{bus}^*$$

{ $Z = R + jX$

$\therefore Z_{bus} = R_{bus} + jX_{bus}$

$$= \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ \vdots & & & \\ r_{n1} & r_{n2} & \cdots & r_{nn} \end{bmatrix} + j \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & & \\ x_{n1} & \cdots & x_{nn} \end{bmatrix}$$

Pn

$$P_L + jQ_L = I_{bus}^T . Z_{bus} I_{bus}^* \longrightarrow eq(1)$$

{ Since $Z_{bus}$ is symetrical,

$Z_{bus}^T = Z_{bus}$ }

The bus current vector $I_{bus}$, can also be written as the sum of a real and reactive component of current vector. ie $I = I_p + jI_q$

$$\therefore I_{bus} = I_{bus_p} + jI_{q\,bus}$$

$$= I_p + jI_q$$

$$\therefore I_{bus} = \begin{bmatrix} I_{p1} \\ I_{p2} \\ \vdots \\ I_{pn} \end{bmatrix} + j \begin{bmatrix} I_{q1} \\ I_{q2} \\ \vdots \\ I_{qn} \end{bmatrix}$$

$\therefore$ On combining all this, in eq$^n$ (1)

$$P_L + jQ_L = [I_p + j I_q]^T [R + jX][I_p - j I_q]$$

On expanding and separating real and reactive parts.

we get $P_L = I_p^T R I_p - I_q^T X I_p + I_p^T X I_q + I_q^T R . I_q$   missing matrix

Since X is symmetric matrix $I_q^T X I_p = I_p^T X I_q$

$$\therefore P_L = I_p^T . R I_p + I_q^T . R . I_q$$

$$= \begin{bmatrix} I_{p1} \\ I_{p2} \\ \vdots \\ I_{pn} \end{bmatrix}^T \begin{bmatrix} R_{11} & R_{12} \cdots & R_{1n} \\ \vdots & & \\ R_{n1} & R_{n2} \cdots & R_{nn} \end{bmatrix} \begin{bmatrix} I_{p1} \\ I_{p2} \\ \vdots \\ I_{pn} \end{bmatrix} + \begin{bmatrix} I_{q1} \\ I_{q2} \\ \vdots \\ I_{qn} \end{bmatrix}^T \begin{bmatrix} R \end{bmatrix} \begin{bmatrix} I_{q1} \\ I_{q2} \\ \vdots \\ I_{qn} \end{bmatrix}$$

$$\therefore P_L = \sum_{\substack{i=1 \\ j=1}}^{n} I_{pi} . R_{ij} . I_{pj} + I_{qi} R_{ij} . I_{qj}$$

dPn

$$P_L = \sum_{i=1}^{n} R_{ij} ( I_{pi} . I_{pj} + I_{qi} . I_{qj} )$$

Transmission losses has been expressed in terms of bus current. In actual plant, the system operators usually know bus powers and voltages. Hence it is more practical to express $P_L$ in terms of power and voltage.

$$P_L = \sum_{\substack{i=1 \\ j=1}}^{n} \alpha_{ij} ( P_i P_j + Q_i Q_j ) + \beta_{ij} ( Q_i P_j - P_i Q_j )$$

→ Exact transmission loss formula

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where $\alpha_{ij} = \frac{r_{ij}}{|v_i||v_j|} \cos(\delta_i - \delta_j)$

$$\beta_{ij} = \frac{r_{ij}}{|v_i||v_j|} \sin(\delta_i - \delta_j).$$

In this cases, even though the formulation for transmission loss is exact, the method requires the calculation of bus impedance matrix which is time consuming and needs more computer memory.

## Modified Coordination Equation.

These equations are derived as follows.

As explained before, the transmission loss is the algebraic sum of all powers in all buses.

ie $$P_L + jQ_L = \sum_{i=1}^{n} S_i$$

$$= \sum_{i=1}^{n} P_i + jQ_i$$

On separating real part,

$$P_L = \sum_{i=1}^{n} P_i$$

$P_L$ is the function of $P_1, P_2 \cdots\cdots$ upto $P_n$

if 'f' is a function of $x_1, x_2 \cdots\cdots x_n$

$$df = \frac{\partial f}{\partial x_1} \cdot dx_1 + \frac{\partial f}{\partial x_2} \cdot dx_2 + \cdots\cdots + \frac{\partial f}{\partial x_n} \cdot dx_n$$

$\therefore$ $\|^{ly}$ $P_L$ is a function of $P_1, P_2, \ldots\ldots P_n$

$$\therefore \quad dP_L = \frac{\partial P_L}{\partial P_1} \cdot dP_1 + \frac{\partial P_L}{\partial P_2} \cdot dP_2 + \cdots\cdots + \frac{\partial P_L}{\partial P_n} \cdot dP_n$$

$$\therefore \quad dP_L = \sum_{i=1}^{n} \frac{\partial P_L}{\partial P_i} \cdot dP_i$$

Let in the interconnected system, bus powers of only two plants $j$ and $n$ be changed by small amounts, keeping the powers at all other buses fixed, then.

$P_L$ is function of '$j$' and '$n$'

$$\therefore \quad dP_{L\,j,n} = \frac{\partial P_L}{\partial P_j} \, dP_j + \frac{\partial P_L}{\partial P_n} \cdot dP_n$$

$\therefore$ change in $P_L$ = change in power at $j$ + change in power at $n$

$$\therefore \quad dP_{L\,j,n} = dP_{Lj} + dP_{Ln}$$

$$\therefore \quad dP_j + dP_n = \frac{\partial P_L}{\partial P_j} \cdot dP_j + \frac{\partial P_L}{\partial P_n} \cdot dP_n$$

ie

$$dP_j \left[ 1 - \frac{\partial P_L}{\partial P_j} \right] + dP_n \left[ 1 - \frac{\partial P_L}{\partial P_n} \right] = 0$$

$$\frac{dP_j}{dP_n} = - \frac{1 - \partial P_L / \partial P_n}{1 - \partial P_L / \partial P_j} \longrightarrow \text{Eq (1)}$$

Since the co-ordination equation is $\frac{dF_i^o}{dP_i^o} \cdot \frac{1}{1-\frac{dP_L}{dP_i^o}} = \lambda$

$\therefore$

$$\frac{dF_j^o}{dP_j^o} \cdot \frac{1}{1-\frac{dP_L}{dP_j^o}} = \lambda$$

$$\frac{dF_n}{dP_n} \cdot \frac{1}{1-\frac{dP_L}{dP_n}} = \lambda$$

$\therefore$

$$\frac{\frac{dF_n}{dP_n}}{\frac{dF_j^o}{dP_j^o}} = \frac{1-\frac{dP_L}{dP_n}}{1-\frac{dP_L}{dP_j^o}}$$

Substitute from eq (1)

$$\frac{dF_n/dP_n}{dF_j^o/dP_j^o} = -\frac{dP_j^o}{dP_n}$$

$$= -\frac{dP_j^o}{dP_{L_{j,n}} - dP_j^o}$$

Since we have $dP_{L_{j,n}} = dP_j^o + dP_n$

$$= -\frac{1}{\frac{dP_{L_{jn}}}{dP_j^o} - 1}$$

$$\frac{dF_n/dP_n}{dF_j^o/dP_j^o} = \frac{1}{1-\frac{dP_{L_{j,n}}}{dP_j^o}}$$

Modified co-ordination equation →

$$\boxed{\frac{dF_n}{dP_n} = \frac{1}{1-\frac{dP_{L_{j,n}}}{dP_j^o}} \cdot \frac{dF_j^o}{dP_j^o}}$$

Here plant 'n' is taken as the reference plant.

From the above expression, it is clear that for economic load dispatch, the condition required is that the incremental cost of power at plant bus n is equal to the incremental cost of power at plant bus j corrected for the effect of the incremental transmission loss involved. These equations are known as modified co-ordination eqn. The modified co-ordination equation can be rewritten as

$$\frac{dF_n}{dP_n} = -\frac{dF_j^{\circ}}{dP_j^{\circ}} \cdot \frac{\Delta P_j^{\circ}}{\Delta P_n} = \mu .$$

# Module VI

## Power System stability.

Reference:
Modern PSA - Kothari
PSA : Nagoorkani.

The stability of an interconnected power system is its ability to return to its normal or stable operation, after having been subjected to some form of disturbances.

Power system stability problems are classified into three basic types - steady state, dynamic and transient.

The study of steady state stability is basically concerned with the determination of the upper limit of machine loadings before losing synchronism, provided the loading is increased gradually.

Dynamic stability is more probable than steady state stability. Small disturbances are continually occuring in a power system, which are small enough not to cause the system to lose synchronism, but do excite the system into the state of natural oscillations. The system is said to be dynamically stable, if the oscillations do not acquire more than certain amplitude and die out quickly.

In a dynamically unstable system, the oscillation amplitude is large and these persists for a long time. (ie the system is underdamped). Dynamic stability can be significantly improved through the use of power system stabilizers.

Following a sudden disturbance on a power system rotor speeds, rotor angular differences and power transfer undergo fast changes, whose magnitudes are depended upon the severity of disturbance For a large disturbance, changes in angular differences may be so large as to cause the machine to fall out of step. This type of Instability is called transient Instability and is a fast phenomenon usually occuring within 1s for a generator close to the cause of disturbance.

Whether the system is stable on occurance of a fault depends not only on the system itself, but also on the type of fault, location of fault, rapidity of clearing and method of clearing. The transient stability limit is almost always lower than the steady state limit, but unlikely it may exhibit different values depending on the nature, location and magnitude of disturbance.

Modern power systems have many interconnected generating stations, each with several generators and many loads. The machines located at any one point in a system normally act in unison. It is therefore a common practice in stability analysis to conside all the machines in one point as one large machine. Also machines which are not separated by lines of high reactance are lumped together and considered as one equivalent machine.

## Dynamics of Synchronous Machines.

Let ;

$J$ = rotor moment of Inertia in $kg.m^2$

$\omega_{sm}$ = angular velocity of rotor in (mech) rad/s.

$\therefore$ Kinetic Energy of the rotor at synchronous machine is

$$K.E = \frac{1}{2} J \omega_{sm}^2 \text{ Joule}$$

$$K.E = \frac{1}{2} J \omega_{sm}^2 \times 10^{-6} \text{ MJ}$$

If $\omega_s$ = velocity of rotor in (elect) rad/s

$P$ = no: of poles of machine;

$$\therefore \quad \omega_s = \left(\frac{P}{2}\right) . \omega_{sm}$$

$$\therefore \quad KE = \frac{1}{2} . J . \left(\frac{2}{P}\right)^2 . \omega_s^2 \times 10^{-6} \text{ MJ}$$

$$= \frac{1}{2} \underbrace{\left[J . \left(\frac{2}{P}\right)^2 . \omega_s \times 10^{-6}\right]}_{M} \omega_s \text{ MJ}.$$

where $M$ = Moment of Inertia in MJ-s / elect rad.

$$\therefore \quad KE = \frac{1}{2} M \omega_s . \text{ MJ}.$$

Define Inertia Constant $H$ = $\dfrac{\text{stored K.E in MJ at synchr speed}}{\text{Machine rating in MVA.}}$

$$H = \frac{\frac{1}{2} M \omega_s}{G}$$

$G \rightarrow$ Machine base MVA.

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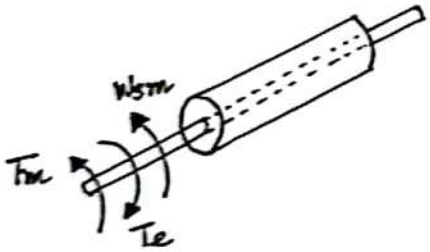
$$M = \frac{2GH}{\omega_s} \qquad f \rightarrow \text{frequency in Hz.}$$

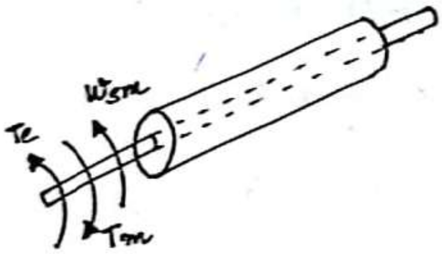
$$= \frac{2GH}{2\pi f} \quad MJ\text{-}s/\text{elect rad.}$$

$$= \frac{GH}{180f} \quad MJ\text{-}s/\text{elect degree}$$

$$\boxed{\frac{M}{G} = \frac{H}{180f} \quad \text{ie} \quad M_{p.u} = \frac{H}{180f}}$$

## Swing Equation.




The rotor of a synchronous machine is subjected to two torques, Te and Tm, which are acting in opposite direction.

where $T_e$ = Net electrical torque in N-m

$T_m$ = Mechanical or shaft torque in N-m.

Under steady state condition, $T_e = T_m$ and the machine runs at constant speed called synchronous speed. If there is any difference between the two, then rotor will have an accelerating or deaccelerating torque, denoted as $T_a$.

$$\therefore \quad T_a = T_m - T_e$$

Let $\theta_m$ = angular displacement of rotor w.r.t stationary reference frame.

$\delta_m$ = angular displacement of rotor w.r.t synchronously rotating reference axis.

By Newton's Second Law; $F = m.a$

$$\therefore \quad T_a = J.\frac{d^2\theta_m}{dt^2}$$

ie $$J\frac{d^2\theta_m}{dt^2} = T_m - T_e \qquad \{\text{Since } T_a = T_m - T_e\}$$

Angular displacement $\theta_m$ and $\delta_m$ are related by the expression,

$$\theta_m = \omega_{sm}.t + \delta_m$$

$$\therefore \quad \frac{d\theta_m}{dt} = \omega_{sm} + \frac{d\delta_m}{dt}$$

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

$$\therefore \quad J\frac{d^2\delta_m}{dt^2} = T_m - T_e$$

Let $P_m$ = mechanical power (shaft power) neglecting losses (MW)

$P_e$ = Electrical power developed in rotor (MW)

$$P = \frac{2\pi NT}{60} \qquad \therefore \quad P_m = \omega_{sm}.T_m$$

$$= \omega T \qquad\qquad P_e = \omega_{sm}.T_e$$

$$\therefore \quad J \cdot \frac{d^2 \delta_m}{dt^2} = \frac{P_m}{\omega_{sm}} - \frac{P_e}{\omega_{sm}}$$

$$\therefore \quad J \cdot \omega_{sm} \cdot \frac{d^2 \delta}{dt^2} = P_m - P_e \longrightarrow (1)$$

Now; the Inertia constant $H = \dfrac{\text{stored K.E}}{\text{Base MVA } (G)}$

$$H = \frac{\frac{1}{2} J \omega_{sm}^2}{G}$$

$$\therefore \quad J \omega_{sm} = \frac{2GH}{\omega_{sm}} \quad \therefore \quad \frac{2GH}{\omega_{sm}} \cdot \frac{d^2 \delta_m}{dt^2} = P_m - P_e$$

$$\left\{ \delta_m = 2/p \cdot \delta \qquad \omega_{sm} = 2/p \, \omega_s \right\}$$

$$\therefore \quad \frac{2GH}{2\pi f} \cdot \frac{d^2 \delta}{dt^2} = P_m - P_e$$

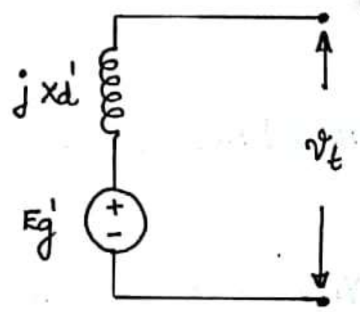
$$\therefore \quad \frac{GH}{\pi f} \cdot \frac{d^2 \delta}{dt^2} = P_m - P_e$$

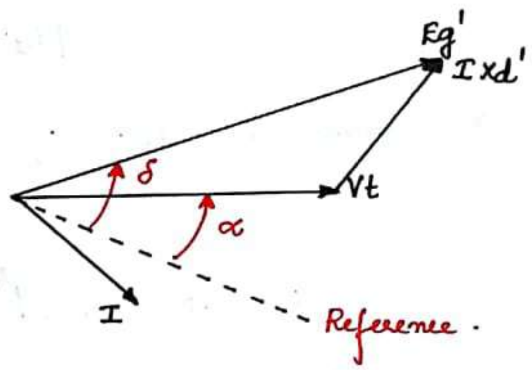
$$\therefore \quad \boxed{\frac{H}{\pi f} \cdot \frac{d^2 \delta}{dt^2} = P_{m\,(p.u)} - P_{e\,(p.u)}}$$

This Equation is called swing equation, which governs the dynamics of synchronous machine rotor. It is a non-linear second order differential equation.

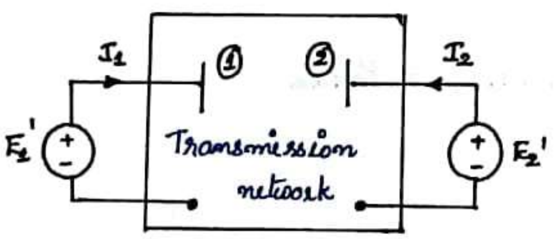
## Power Angle Equation.

The equation relating the electrical power generated in a synchronous machine to the angular displacement of the rotor ($\delta$) is called power angle equation. Power angle equation can be derived using the transient model of the generator.




Let a single generator supplies power through a transmission s/m to a load or a large system at other end. Such a system can be represented by a 2 bus network as shown below.




$E_1'$ = Transient internal voltage of generator at bus 1

$E_2'$ = Voltage at the receiving end.

We know that $I_{bus} = Y_{bus} \cdot V_{bus}$

for the network shown;

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} E_1' \\ E_2' \end{bmatrix}$$

$$\therefore \quad I_1 = Y_{11} E_1' + Y_{12} E_2'$$

Complex power at bus 1 $= P_1 + jQ_1$

$$= VI^*.$$

$$\therefore \quad P_1 + jQ_1 = E_1'(I_1)^*$$

$$= E_1'\left[E_1' Y_{11} + Y_{12}. E_2'\right]^*$$

$$= E_1'\left[E_1'^* Y_{11}^* + Y_{12}^* E_2'^*\right]$$

$$= |E_1'|^2 \cdot Y_{11}^* + E_1' E_2'^* Y_{12}^*.$$

We know; $E_1' = |E_1'| \angle \delta_1$ ; $E_2' = |E_2'| \angle \delta_2$ ; $Y_{11} = |Y_{11}| \angle \theta_{11}$ ; $Y_{12} = |Y_{12}| \angle \theta_{12}$

$$\therefore \quad P_1 + jQ_1 = |E_1'|^2 Y_{11} \angle -\theta_{11} + |E_1'||E_2| \, |Y_{12}| \angle \delta_1 - \delta_2 - \theta_{12}.$$

On separating real and imaginary parts.

$$P_1 = |E_1'|^2 |Y_{11}| \cos(\theta_{11}) + |E_1'| \, |E_2'| \, |Y_{12}| \cos(\delta_1 - \delta_2 - \theta_{12})$$

$$P_1. = |E_1'|^2 \cdot G_{11} + |E_1'| \, |E_2'| |Y_{12}| \cos(\delta_1 - \delta_2 - \theta_{12})$$

Let $|E_1'|^2 \cdot G_{11} = P_c$ ; $|E_1'| \, |E_2'| \, |Y_{12}| := P_{max}$

$$\delta_1 - \delta_2 = \delta$$

$$\theta_{12} = \pi/2 + \gamma$$

$$\therefore \quad P_1 = P_c + P_{max} \cos(\delta - \pi/2 + \gamma)$$

$$P_1 = P_c + P_{max} \sin(\delta - \gamma)$$

Power angle equation

ie $\boxed{P_e = P_c + P_{max} \sin(\delta - \gamma)}$

where

$P_c$ = power loss in s/m

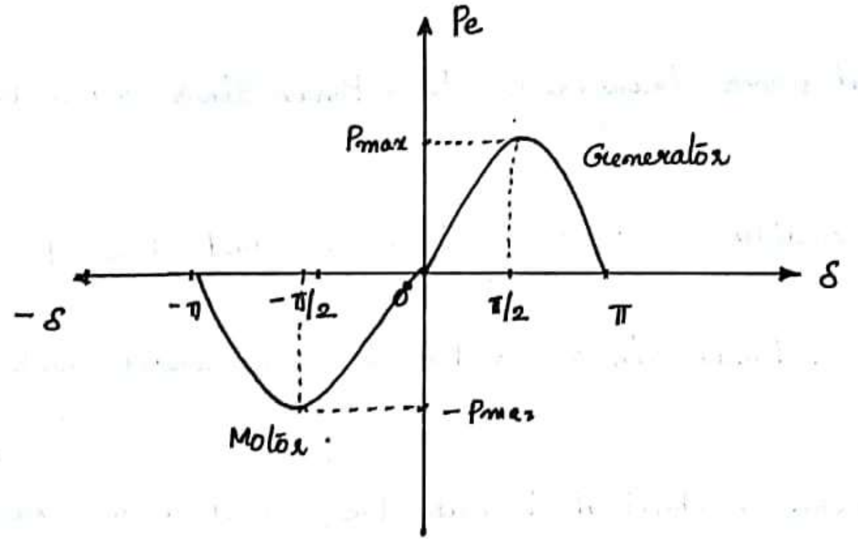
$P_{max}$ = max real power that can be transfered

For a purely reactive network $P_c = 0$ since $G_{11} = 0$ ; $\theta_{12} = 90^\circ$
$\therefore \gamma = 0^\circ$

$$\boxed{\begin{aligned} \therefore \quad P_e &= P_{max} \sin \delta \longrightarrow (1) \\ \text{where} \quad P_{max} &= |E_1'||E_2'|.|Y_{12}| \\ &= \frac{|E_1'||E_2'|}{X_{12}} \end{aligned}}$$

This Equation (1) is called simplified power angle equation.

The plot or graph of Pe as a function of '$\delta$' is called power angle curve




We have Swing Equation $\frac{H}{\pi f} \cdot \frac{d^2\delta}{dt^2} = P_m - P_e$ ; On substituting $P_e$ from power angle equation; we get

$$\frac{H}{\pi f} \cdot \frac{d^2\delta}{dt^2} = P_m - P_{max} \sin \delta$$

## Steady state stability.

In steady state, every synchronous machine has a limit for power transfer to a receiving system. Steady state limit of a machine or a transmitting system is the maximum power that can be transmitted to a receiving system, without loss in synchronism.

Let $|E|$ = magnitude of steady state internal Emf of the machine

$|V|$ = magnitude of voltage at receiving end.

$X$ = Transfer reactance between synchronous machine and receiving system.

Then the real power transfered $P_e = P_{max} \sin \delta$ where $P_{max} = \dfrac{|E||V|}{X}$

Let;

Under ideal condition $\delta = \delta_0$ ; $P_e = P_{e0}$ and $P_m = P_{e0}$.

$\therefore \quad P_{e_0} = P_{max} \sin \delta_0 = P_m \; \left\{ \text{Since under stable cond}^n \; P_m = P_e \right\}$

Let with the same mechanical input $P_m$, load angle changed to $\Delta\delta$

$\therefore$ Electrical power generated changes by $\Delta P$.

$\therefore \quad P_e = P_{e_0} + \Delta P$

new $\quad P_e = P_{max} \sin(\delta_0 + \Delta\delta)$

$$= P_{max}\left[\sin \delta_0 \cos \Delta\delta + \cos \delta_0 \sin \Delta\delta\right]$$

Since $\Delta\delta$ is very small $\sin \Delta\delta \simeq \Delta\delta$ and $\cos \Delta\delta \simeq 1$

$\therefore \quad P_e = P_{max} \cdot \sin\delta_0 \cdot 1 + P_{max} \cos \delta_0 \, \Delta\delta$

ie $\quad P_{e_0} + \Delta P = \underbrace{P_{max} \sin \delta_0}_{P_{e_0}} \cdot \delta + P_{max} \cos \delta_0 \, \Delta\delta$

$$\therefore \quad \Delta P = P_{max} \cos \delta_0 \cdot \Delta\delta$$

Now; as per the swing equation; $\frac{H}{\pi f} \cdot \frac{d^2\delta}{dt^2} = P_m - P_e$

or

$$M \cdot \frac{d^2\delta}{dt^2} = P_m - P_e$$

for a change in load angle $\Delta\delta$;

$$M \frac{d^2}{dt^2}(\delta_0 + \Delta\delta) = P_m - (P_{e_0} + \Delta P) \qquad \because \text{ Since } P_m = P_{e_0}$$

$$= -\Delta P$$

ie $\quad M \cdot \frac{d^2}{dt^2}(\delta_0 + \Delta\delta) = -P_{max} \cos \delta_0 \cdot \Delta\delta \quad ; \quad \delta_0 \rightarrow$ constant (since initial value).

$$\therefore \quad M \frac{d^2}{dt^2} \cdot \Delta\delta = -(P_{max} \cos \delta_0) \cdot \Delta\delta$$

$$\underbrace{M \frac{d^2}{dt^2}}_{x^2} \Delta\delta + \underbrace{P_{max} \cos \delta_0}_{C} \cdot \Delta\delta = 0$$

$\therefore \quad (M x^2 + C) \, \Delta\delta = 0 \quad ;$ Since $\Delta\delta \neq 0 \quad M x^2 + C = 0$

$$x = \sqrt{-C/M}$$

where $x$ = roots of the characteristic equation.

Case 1: when c is positive (ie $P_{max} \cos \delta_0 > 0$)

when c is +ve; the roots of the equation are purely imaginary. System behaviour is purely oscillatory. In this analysis resistances were neglected. If they are also included, the roots will be complex conjugate and the s/m will be stable.

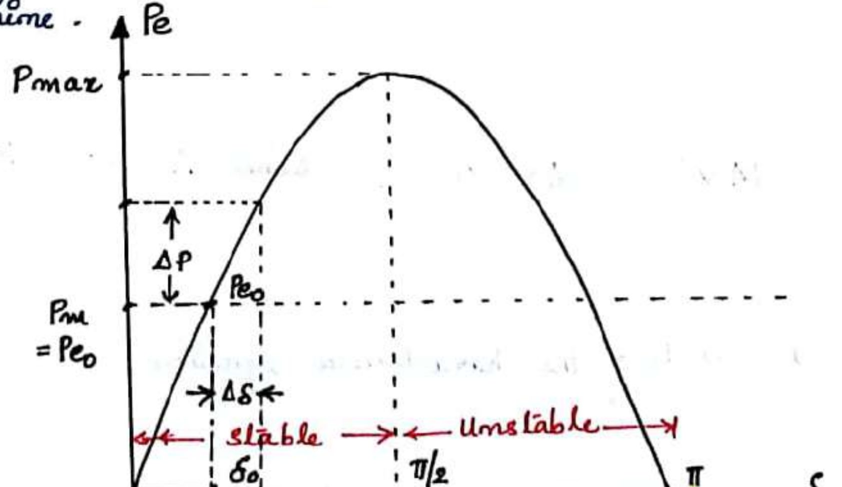
∴ The practical system is stable for small increment in power; provided $P_{max} \cos \delta_0 > 0$.

Case 2: when c is negative ($P_{max} \cos \delta_0 < 0$)

when c is -ve, the roots of the equation are real and equal in magnitude. Due to the +ve root, the torque angle increases without bound and finally lose synchronism.

Steady state limit.

The term $P_{max} \cos \delta_0$ denotes the steady state stability of the system. Hence it is called synchronising coefficient or stiffness of synch. machine.



when $0 < \delta_0 < \pi/2$ $\quad \cos\delta_0 > 0$

ie $P_{max}\cos\delta_0 > 0$ ie; the system is stable.

when $\delta_0 = \pi/2$

$P_{max}\cos\delta_0 = 0$ and $P_e = P_{max}$

when $\pi/2 < \delta_0 < \pi$ $\quad \cos\delta_0 < 0$

$\therefore$ $P_{max}\cos\delta_0 < 0$ ; ie system unstable.

$\therefore$ The machine will operate in stable operating condition for the load angle or torque angle $0 < \delta < \pi/2$. Practically the system has to be operated below the steady state stability limit.

## TRANSIENT STABILITY

The transient stability of a system is concerned with the study of s/m behaviour for large disturbances. The short circuits and switching heavy loads can be treated for this case. The dynamics of the system under transient state is governed by the non-linear swing equation $\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e$.

The transient stability of a single machine connected to an infinite bus can be determined using a simple criteria called equal area criteria.

## EQUAL AREA CRITERIA.

The transient stability analysis of a simple system can be performed by using equal area criteria.

During transient state of a power system, there are two situations for change in $\delta$ (torque angle or load angle) with respect to time.

(i) The '$\delta$' may increase to a maximum value and then decrease to a stable value. Then the system is considered as stable.

(ii) The '$\delta$' may keep on increasing indefinitely. In this case, system is unstable.

ie; for s/m to be stable $\frac{d\delta}{dt} = 0$ at some time instant

s/m is unstable if $\frac{d\delta}{dt} > 0$ for long time.

Consider the swing equation of generator connected to infinite bus bar.

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_m - P_e$$

$P_m - P_e = P_a$ Accelerating or decelerating power.

$$\therefore \frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_a$$

or

$$M \cdot \frac{d^2\delta}{dt^2} = P_a$$

$\left\{ \text{where } M = \frac{H}{\pi f} = \text{moment of Inertia} \right\}$

$$\therefore \frac{d^2\delta}{dt^2} = \frac{P_a}{M}$$

On Multiplying both sides with $2 \cdot \frac{d\delta}{dt}$

$$\therefore 2 \frac{d\delta}{dt} \cdot \frac{d^2\delta}{dt^2} = 2 \cdot \frac{d\delta}{dt} \cdot \frac{P_a}{M}$$

$$2 \cdot \frac{d\delta}{dt} \frac{d^2\delta}{dt^2} = \frac{2}{M} \cdot \frac{d\delta}{dt} \cdot P_a$$

$$2 \cdot \frac{d\delta}{dt}\left(\frac{d^2\delta}{dt^2}\right) = \frac{2 \cdot P_a}{M} \frac{d\delta}{dt}$$

On Integrating both sides w.r.t $d\delta$

$$2\int \frac{d^2\delta}{dt^2} \cdot d\delta = \frac{2}{M}\int P_a \cdot d\delta$$

$$= \left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}\int_{\delta_0}^{\delta} P_a \cdot d\delta$$

or $$\frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int_{\delta_0}^{\delta} P_a \cdot d\delta}$$

for the s/m to be stable $\frac{d\delta}{dt} = 0$ ie $\int_{\delta_0}^{\delta} P_a \cdot d\delta = 0$

or $$\int_{\delta_0}^{\delta} (P_m - P_e) \cdot d\delta = 0$$

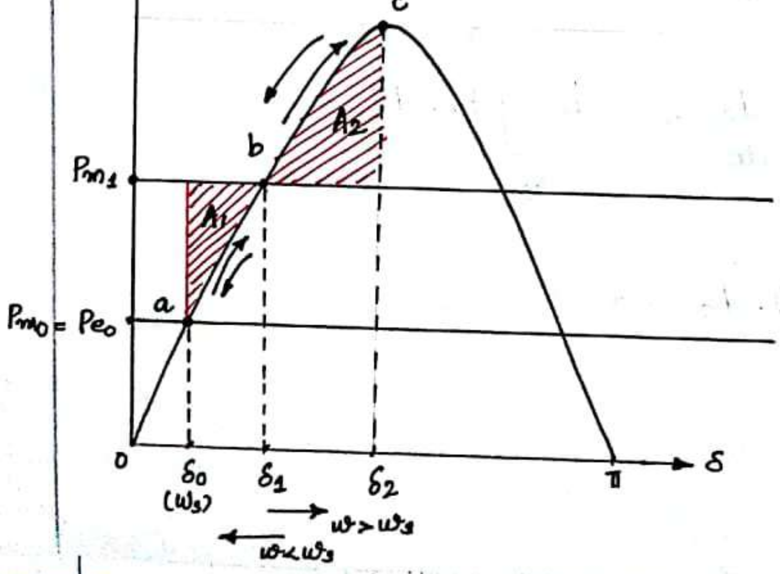
The condition for stability can therefore be stated as: The system is stable if, the area under $P_a - \delta$ curve reduces to zero at some value of $\delta$. In other words, the +ve (accelerating) area under $P_a - \delta$ curve must equal the −ve (decelerating) area under $P_a - \delta$ curve and hence the name equal area criterion of stability.

## Sudden change in Mechanical Input.

Figure shows the transient model of a single machine tied to infinite bus bar. The electrical power transmitted is given by


$$P_e = P_{max} \sin\delta \quad ; \quad P_{max} = \frac{|E'||V|}{X'_d + X_e}$$

Under steady operating condition, $P_{m_0} = P_{e_0} = P_{max} \sin\delta$. This is indicated by point 'a' in the $P-\delta$ curve (power angle curve).




Let the mechanical input to the rotor is suddenly increased from $P_{m0}$ to $P_{m1}$. Now the power $P_a = P_{m1} - P_e$ is +ve and causes the rotor to accelerate and so does the rotor angle. As the angle increases from $\delta_0$, the operating region also shifts from point 'a'. At angle $\delta_1$, $P_{m1} = P_e$. ie $P_a = 0$, a stable state is reached, but the rotor angle continue to increase due to the inertia of the machine. Hence the operating region again shifts from 'b' along the P-δ curve. Pa now becomes negative, and the rotor starts decelerating and let at $\delta_2$, at point 'c', the accelerating area $A_1$ equals deacceleration area $A_2$. Due to deceleration, the rotor speed decreases and so does the rotor angle and the operating region travels back in the P-δ curve and finally settle down at new steady state $\delta_1$, where $P_{m1} = P_e$.

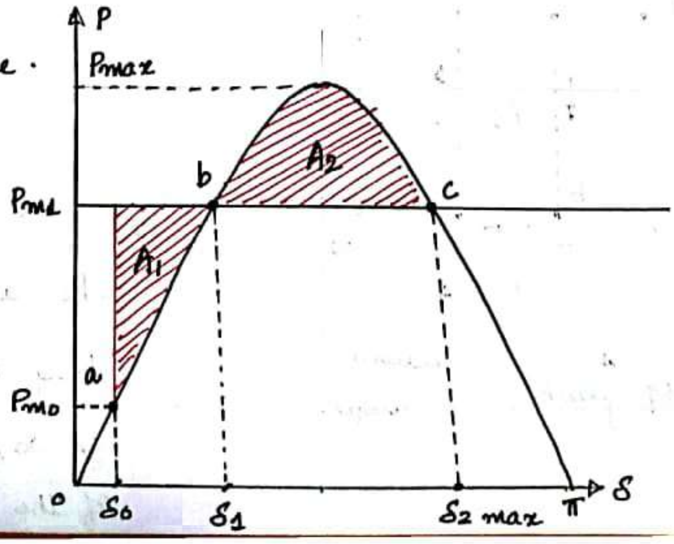
From the figure, $$A_1 = \int_{\delta_0}^{\delta_1} (P_{m1} - P_e) \cdot d\delta$$

$$A_2 = \int_{\delta_1}^{\delta_2} (P_e - P_{m1}) \cdot d\delta .$$

For the system to be stable, it should be possible to find the value $\delta_2$, such that $A_1 = A_2$. As $P_m$ is increased, a limiting condition is finally reached, where $A_1$ equals the area $A_2$ as shown. Under this condition, $\delta_2$ acquires the maximum value.

$$\therefore \delta_2 = \delta_{max} = \pi - \delta_1$$

$$= \pi - \sin^{-1}\left(\frac{P_{m1}}{P_{max}}\right)$$

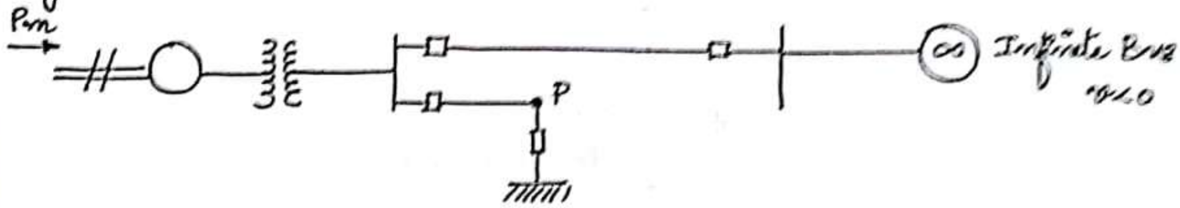


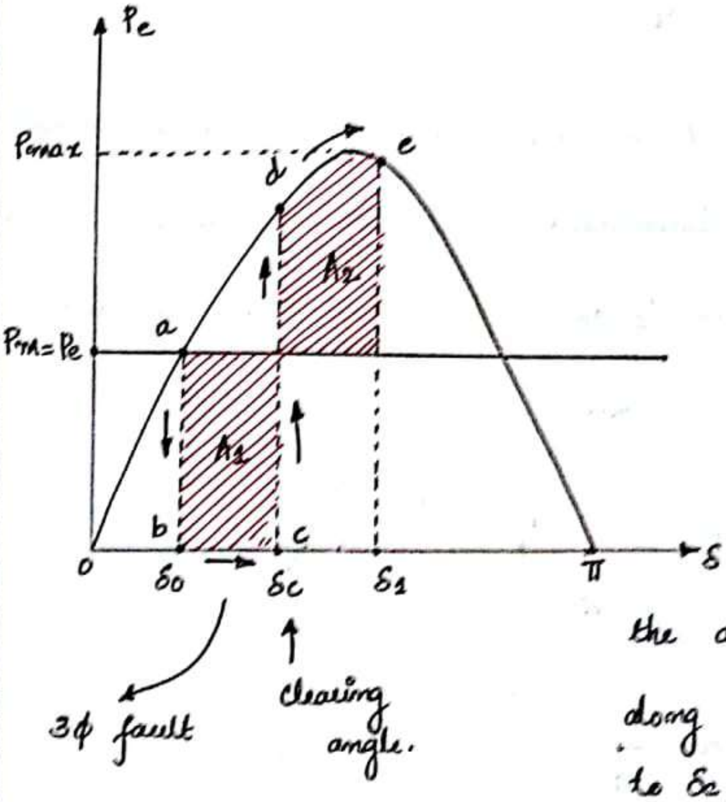
Any further increase in $P_m$, will reduce the area $A_2$ and which violates the equal area criterion of transient stability.

It may be also noted that; even when the rotor angle is increased beyond $\delta = 90°$, the system can be transiently stable, as long as equal area criteria is met. The condition $\delta = 90°$ is meant for steady state stability and does not apply to transient stability case.

## Effect of clearing time on stability.

Let the system shown in figure be operating with mechanical input $P_m$ at a steady angle of $\delta_0$ ($P_m = P_e$) as shown by point 'a' on the $P_e$-$\delta$ diagram.




If a 3$\phi$ fault occurs at the point P of the outgoing radial line, the electrical output of the machine will reduce to zero. ie $P_e = 0$, hence the operating region shifts from 'a' to 'b'. Now $P_a = P_m - P_e$ is +ve and the rotor accelerates and $\delta$ increases. ie, the accelerating area $A_1$ starts increasing along bc. Let at time $t = t_c$ corresponding to $\delta_c$, the fault is cleared, by the opening of the circuit breaker.

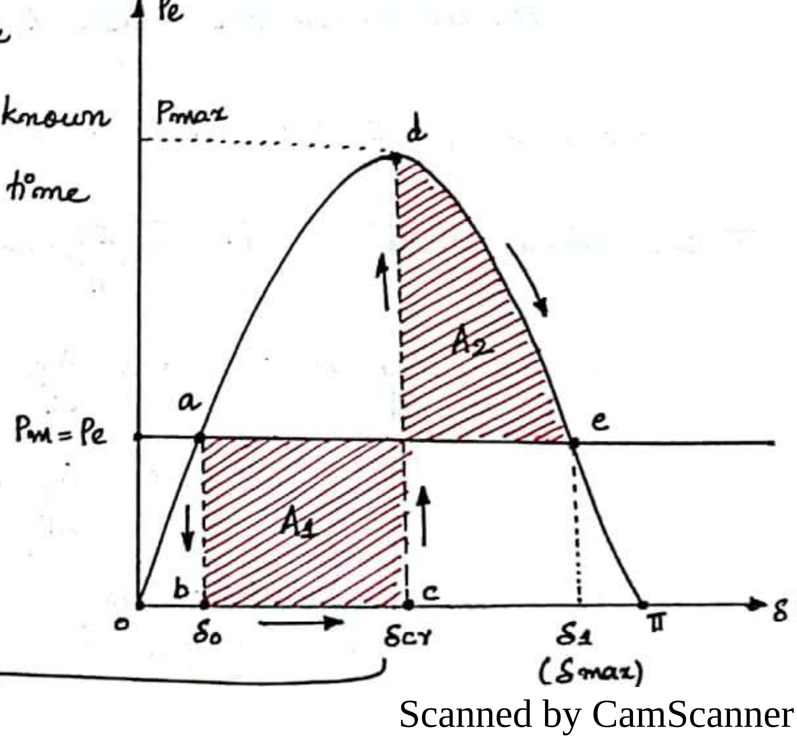
The system once again becomes healthy and transfer power $P_e = P_{max} \sin\delta$. Hence at '$\delta_c$' the operating point shifts from 'c' to 'd' along the P-$\delta$ curve. Now $P_a = P_m - P_e$ is negative and the rotor decelerates and the decelerating area $A_2$ starts along 'de'. If at an angle $\delta_1$, $A_1 = A_2$, the system is found to be stable. The system finally settles at the steady operating point 'a' in an oscillatory manner due to inherent damping.

The values of $t_c$ and $\delta_c$ are called clearing time and clearing angle respectively.

Imp

## Critical clearing angle and Critical clearing time.

In the above system mentioned, as the clearing of fault is delayed, the area $A_1$ goes on increases and hence $\delta_1$ also increases. $\delta_1$ can be increased only upto $\delta_{max}$. For the system to be stable, $A_1 = A_2$. Let in the power angle curve shown below, a clearing angle $\delta_{cr}$ is mentioned, beyond which $A_2 < A_1$. The maximum allowable value of the clearing time and angle for the system to remain stable are known respectively as critical clearing time and critical clearing angle.




Critical clearing angle

For the case mentioned; ie during fault, if $P_e = 0$, the relation of critical clearing angle and time is derived here. All angles are in radians.
(note: For other cases, the equations may be different depending on fault).

Referring to the power angle curve:-

$\delta_{max} = \pi - \delta_0$ ; $P_e = P_{max} \sin\delta$ ; at $\delta_0$; $P_e = P_m$ ; $\therefore P_m = P_{max} \sin\delta_0$

$\therefore \delta_0 = \sin^{-1}\left(\frac{P_m}{P_{max}}\right)$ ie; $\delta_{max} = \pi - \sin^{-1}(P_m/P_{max})$.

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - P_e).d\delta \qquad \{\because P_e = 0\}$$

$$= \int_{\delta_0}^{\delta_{cr}} P_m.d\delta = P_m \delta \Big\}_{\delta_0}^{\delta_{cr}}$$

$$= P_m \{\delta_{cr} - \delta_0\}$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_e - P_m).d\delta$$

$$= \int_{\delta_{cr}}^{\delta_{max}} P_{max} \sin\delta - \int_{\delta_{cr}}^{\delta_{max}} P_m . d\delta$$

$$= -P_{max} \cos\delta \Big\}_{\delta_{cr}}^{\delta_{max}} - P_m \delta \Big\}_{\delta_{cr}}^{\delta_{max}}$$

$$= -P_{max}\{\cos\delta_{max} - \cos\delta_{cr}\} - P_m\{\delta_{max} - \delta_{cr}\}$$

For the system to be stable $A_1 = A_2$; On Equating two areas,

$$\boxed{\delta_{cr} = \cos^{-1}\left[(\pi - 2\delta_0).\sin\delta_0 - \cos\delta_0\right]}$$

As per swing equation; $\frac{d^2\delta}{dt^2} = \frac{\pi f}{H}.P_m$ {Since $P_e = 0$}.

On Integrating twice; $\delta = \frac{\pi f}{2H} P_m t^2 + \delta_0$ if $\delta = \delta_{cr}$, $t = t_{cr}$

ie $\delta_{cr} = \frac{\pi f}{2H} P_m t_{cr}^2 + \delta_0$

$$\boxed{\text{or} \quad t_{cr} = \sqrt{\frac{2H(\delta_{cr} - \delta_0)}{\pi f P_m}}}$$

where $\delta_{cr}$ = critical clearing angle
$t_{cr}$ = critical clearing time.

## Factors affecting Transient stability and Methods for Improving the same.

Transient stability mainly depends on the type and location of the fault.

As $M.\frac{d^2\delta}{dt^2} = P_m - P_e$ {from swing equation}

$$\frac{d^2\delta}{dt^2} = \frac{P_m - P_e}{M}$$

An increase in the moment of inertia M reduces the angle through which rotor swings in a given time interval. Hence stability can be improved by increasing M. But it cannot be practical due to economic reasons. Also increasing M will have an undesirable effect of slowing down the response of speed governor loop.

### Methods of Improving transient stability limit: (May 2016 – 4 marks)

1. Increase of system voltages, use of AVR (Automatic Voltage Regulators)
2. Use of high speed excitation systems.
3. Reduction in system transfer reactance
4. Use of high speed reclosing breakers.

During fault, the reduction in system voltage can be automatically sensed by AVR which helps to restore the generator voltage.

The transfer reactance can be reduced to improve the stability limit. This can be done by (i) reducing conductor spacing or

(ii) by increasing conductor diameter. Compensation for line reactance by series capacitor is an effective and economic method of increasing stability limit specially for transmission distance of more than 350 km. Transfer reactance can also be reduced by increasing the number of parallel line between transmission points. Rapid switching and isolation of unhealthy lines followed by reclosing also improves stability margins.

Recent methods of Improving stability are:-

HVDC Links: Increased use of HVDC links employing thyristors would deviate stability problem. There is no risk of a fault in one system causing loss of stability in the other system.

Breaking Resistors: For improving stability where clearing is delayed or large load is suddenly lost, a resistive load called a breaking resistor is connected at or near the generator bus. This load compensates for some of the reduction of load on generators.

Bypass Valving: In this method, the stability of a unit is improved by decreasing the mechanical input power to the turbine.

Full Load Rejection Technique: Fast valving combined with high-speed clearing time will be sufficient to maintain stability in most cases. A full load rejection technique could be utilized after the unit is separated from the system. To do this, the unit has to be equipped with a large steam bypass system. The main disadvantage is the extra cost of large bypass system.

Multimachine stability : Refer Modern PSA : Kothari.
Chapter Power system stability : Pg no: 455

# TUTORIALS

1. A 50 Hz, four pole turbogenerator rated 100 MVA, 11 kV has an inertia constant of 8.0 MJ/MVA.

(a) Find the stored energy in the rotor at synchronous speed.

(b) If the mechanical input is suddenly raised to 80 MW for an electrical load of 50 MW, find rotor acceleration, neglecting mechanical and electrical losses.

(c) If the acceleration calculated in part b is maintained for 10 cycles, find change in torque angle and rotor speed in revolutions/minute at the end of this period.

(MAY 2015 - 7 marks).

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Soln: given,

$f = 50\text{ Hz}$, $P = 4$, $H = 8\text{ MJ/MVA}$, $G = 100\text{ MVA}$

(a) Stored Energy = K.E of rotor

$\therefore$ Stored Energy $= H.G$

$= 8 \times 100$

$= \underline{\underline{800\text{ MJ}}}$

we have

$$H = \frac{\text{Stored Energy (KE)}}{\text{Rated MVA}}$$

(b) $P_m = 80\text{ MW}$ $P_e = 50\text{ MW}$, $\frac{d^2\delta}{dt^2} = ?$ (Rotor acceleration)

we have

$$M.\frac{d^2\delta}{dt^2} = P_m - P_e \quad \text{(from swing equation)}$$

$M(p.u) = \frac{H}{\pi f} = \frac{H}{180 f}$ ; $M = \frac{GH}{180 f}$ { where M is not in p.u. f in Hz }.

$\therefore$ $M = \frac{100 \times 8}{180 \times 50} = 0.0888$ MJ-s/elect degree

$\therefore$ $\frac{d^2\delta}{dt^2} = \frac{P_m - P_e}{M}$

$= \frac{80 - 50}{0.0888} = \underline{\underline{337.83}}$ elect degree/$s^2$

(c) 10 cycles = 0.2 s { Since $f = 50 Hz$ ; for one cycle $\rightarrow$ 20 ms

we have $\frac{d^2\delta}{dt^2} = 337.83$.

$d^2\delta = 337.83 \times (dt)^2$

Change in torque angle } $\therefore$ $d\delta = \frac{1}{2} \cdot 337.83 \times (0.2)^2$

$= \underline{\underline{6.75 \text{ elect degrees}}}$.

2. A 50 Hz, 4 pole turbogenerator rated 40 MVA, 11 kV has an inertia constant of 15kW-s per kVA. Determine the KE stored in the rotor at synchronous speed. Determine the acceleration and accelerating torque, if the shaft input less the rotational losses is 20 MW and electrical power developed is 15 MW. (MAY 2016 - 8 marks)

soln: given,

$f = 50\,Hz$, $P = 4$, $G = 40\,MVA$, $H = 15\,MJ/MVA$

$$KE \text{ stored in the rotor} = G \cdot H = 40 \times 15 = \underline{\underline{600\,MJ}}$$

To find acceleration ; $\frac{d^2\delta}{dt^2} = \frac{P_m - P_e}{M}$

$P_m = 20\,MW$, $P_e = 15\,MW$

$$M = \frac{GH}{180 \cdot f} = \frac{40 \times 15}{180 \times 50} = 0.0666\ MJs/\text{elect degree}.$$

$$\therefore \frac{d^2\delta}{dt^2} = \frac{20 - 15}{0.0666} = \underline{\underline{75.07\ \text{elect degree}/s^2}}$$

$$\text{Accelerating torque} = \frac{\text{Accelerating Power}}{\omega_{sm}}$$

$$= \frac{P_m - P_e}{2\pi f \cdot (2/P)} = \frac{20-15}{2\pi \cdot 50 \cdot 2/4}$$

$$= \underline{\underline{0.55\ N\text{-}m}}$$

Scanned by CamScanner

3. A generator operating at 50 Hz delivers 1 p.u power to an infinite bus through a transmission circuit in which resistance is ignored. A fault takes place reducing the max power transferable to 0.5 p.u whereas before the fault, this power was 2.0 pu and after the clearance of the fault, it is 1.5 p.u. By the use of equal area criteria, determine the critical clearing angle. (November 2015).

Let case I → Pre fault

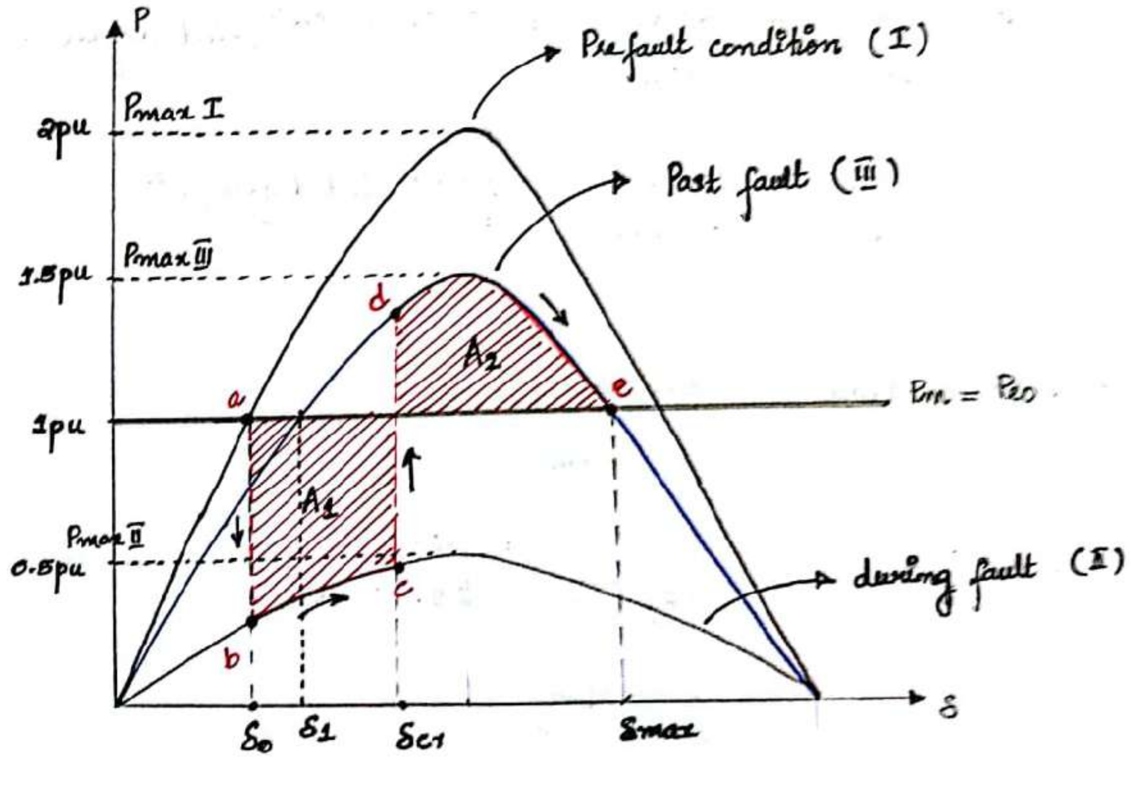
II → During fault

III → Post fault (after fault cleared).

∴ given, $f = 50$ Hz ; $P_{e_0} = 1$ p.u ie $P_m = 1$ p.u

$P_{max\,I} = 2.0$ pu, $P_{max\,II} = 0.5$ pu and $P_{max\,III} = 1.5$ pu

The power angle curves for the above conditions can be drawn as;



Let the s/m was operating under stable condition at point 'a'; where $P_m = P_e = 1\,p.u$ in the I curve. Suddenly a fault occurs and the operating region falls from 'a' to 'b' in the II curve. Let at $\delta_{cr}$, the fault is cleared at point 'c' and operating region shifts to d in curve III.

For the clearing angle to be critical, Area $A_1$ = Area $A_2$.

$$\text{Area } A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - P_{e_{II}}).\, d\delta \qquad A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{e_{III}} - P_m).\, d\delta .$$

Since $P_e = P_{max} \sin\delta$ (general expression)

$P_{e_{II}} = P_{max_{II}} \sin\delta$ and $P_{e_{III}} = P_{max_{III}} \sin\delta$.

$\delta_{max} = \pi - \delta_1$ {from the power angle curve}.

$$\therefore \quad P_{max_{II}} \sin\delta_1 = 1\,p.u$$

$$\therefore \quad \delta_1 = \sin^{-1}(1/P_{max_{III}}) = \sin^{-1}(1/1.5)$$

$$= \underline{\underline{41.8^\circ}} = \underline{\underline{0.729\,rad}}$$

$$\therefore \quad \delta_{max} = \pi - \delta_1$$

$$= 180 - 41.8 = \underline{\underline{138.18^\circ}} = \underline{\underline{2.411\,rad}}$$

$P_{max_I} \sin\delta_0 = 1$

$\delta_0 = \sin^{-1}(1/P_{max_I})$

$= \sin^{-1}(1/2)$

$= \underline{\underline{30^\circ}} = \underline{\underline{0.523\,rad}}$

$$\therefore \text{ Area } A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - P_{e\,II})\, d\delta$$

$$= \int_{\delta_0}^{\delta_{cr}} P_m - \int_{\delta_0}^{\delta_{cr}} P_{max\,II} \sin\delta$$

$$= P_m \cdot \delta \Big\}_{\delta_0}^{\delta_{cr}} + P_{max\,II} \cos\delta \Big\}_{\delta_0}^{\delta_{cr}}$$

$$= \{\delta_{cr} - \delta_0\} + 0.5\{\cos\delta_{cr} - \cos\delta_0\}$$

$$= \{\delta_{cr} - 0.523\} + 0.5\{\cos\delta_{cr} - 0.866\}$$

$$= \delta_{cr} - 0.523 + 0.5\cos\delta_{cr} - 0.433$$

$$= \underline{\underline{\delta_{cr} + 0.5\cos\delta_{cr} - 0.956}}$$

$$\text{Area } A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{e\,III} - P_m)\, d\delta$$

$$= \int_{\delta_{cr}}^{\delta_{max}} P_{max\,III} \sin\delta\, d\delta - \int_{\delta_{cr}}^{\delta_{max}} P_m\, d\delta$$

$$= -1.5\cos\delta \Big\}_{\delta_{cr}}^{\delta_{max}} - \delta \Big\}_{\delta_{cr}}^{\delta_{max}}$$

$$= -1.5\{\cos\delta_{max} - \cos\delta_{cr}\} - \{\delta_{max} - \delta_{cr}\}$$

$$= -1.5\{-0.745 - \cos\delta_{cr}\} - \{2.41 - \delta_{cr}\}$$

$$= 1.117 + 1.5\cos\delta_{cr} - 2.41 + \delta_{cr}$$

$$= \underline{\underline{\delta_{cr} + 1.5\cos\delta_{cr} - 1.293}}$$

On Equating both Areas;

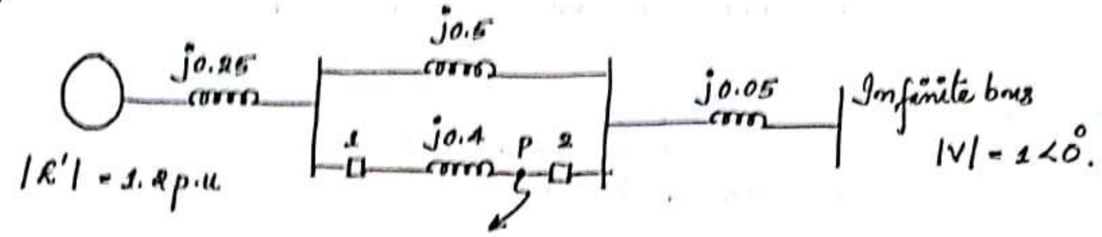
$$\cancel{\delta_{cr}} + 0.5\cos\delta_{cr} - 0.956 = \cancel{\delta_{cr}} + 1.5\cos\delta_{cr} - 1.293$$

$$\therefore \quad -\cos\delta_{cr} = -0.337$$

$$\therefore \delta_{cr} = \cos^{-1}(0.337) = \underline{\underline{70.3^\circ}} \text{ or } 1.22 \text{ radians.}$$

Scanned by CamScanner

4. Consider the system shown in figure, where a three phase fault is applied at the point P as shown.



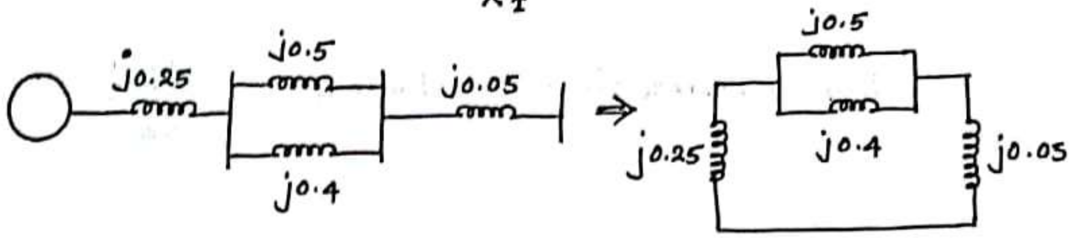

Find the critical angle for clearing the fault with simultaneous opening of the breakers 1 and 2. The reactance values of various components are indicated on the diagram. The generator is delivering 1.0 p.u power at the instant preceding the fault.

sol:

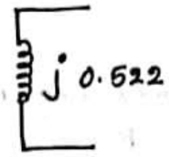
We need to analyse 3 condition :- Prefault, during fault, Post fault.

I. Normal operation or Pre-fault condition :

$Pe_I = P_{max_I} \sin\delta$ ; $P_{max_I} = \dfrac{|E||V|}{X_I}$




Effective X ⇒




$\therefore \quad P_{max_I} = \dfrac{1.2 \times 1}{0.522}$

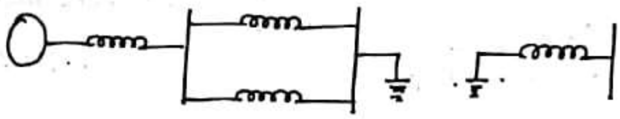
$= \underline{2.29 \text{ p.u}}$

$\therefore \quad Pe_I = 2.29 \sin\delta$

## II During fault.

During fault, no power is transfered.

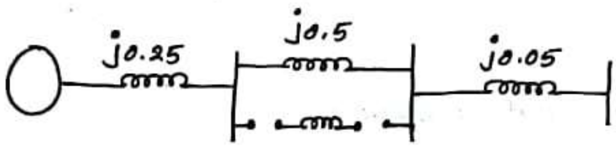
ie



ie $Pe_{II} = 0$

## III Post fault condition (After the fault is cleared).

Here the fault is cleared by opening the circuit breakers 1 and 2 simultaneously. Hence the equivalent circuit will be



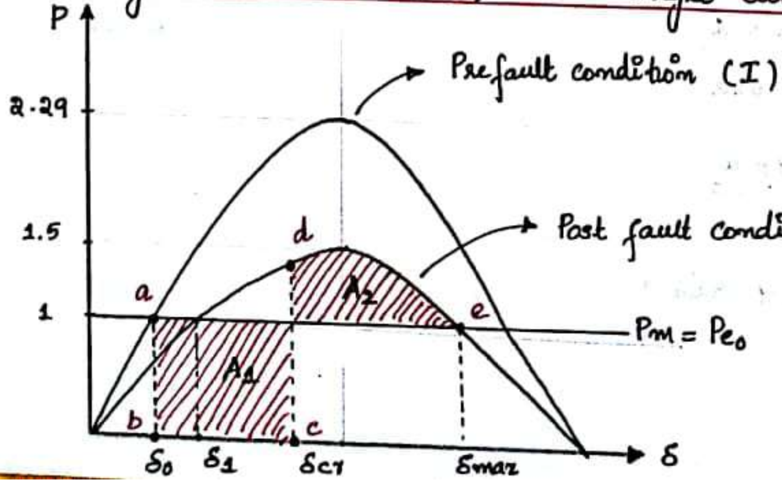

The effective $X = j0.25 + j0.5 + j0.05 = 0.8j$.

$Pe_{III} = P_{max\,III} \sin\delta$ ; $P_{max\,III} = \frac{|E||V|}{X_{III}} = \frac{1.2 \times 1}{0.8}$

$= \underline{\underline{1.5\ p.u}}$

$\therefore\ Pe_{III} = 1.5 \sin\delta$.

On combining all the conditions, Power angle curve can be drawn as:



Let the system was operating under stable condition at point 'a'; where $P_m = P_e = 1$ p.u in the I curve. Suddenly a fault occurs and the operating region shifts from 'a' to 'b'. The load angle continue to increase and let at $\delta_{cr}$, the fault is cleared and operating region shifts from 'c' to 'd' in curve III.

For the fault clearing angle to be critical; the Accelerating area = deaccelerating area

ie Area $A_1$ = Area $A_2$

$$\text{Area } A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - P_{e\,II}) \cdot d\delta \quad \text{and} \quad \text{Area } A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{e\,III} - P_m) \cdot d\delta .$$

$P_{e\,II} = 0$

$P_{max\,I} \sin \delta_0 = 1$

$$\therefore \delta_0 = \sin^{-1}(1/P_{max\,I})$$

$$= \sin^{-1}(1/2.29)$$

$$= \underline{\underline{0.451 \text{ rad}}}$$

$P_{e\,III} = P_{max\,III} \sin \delta$

$= 1.5 \sin \delta$

$\delta_{max} = \pi - \delta_1$

$$\delta_1 = \sin^{-1}(1/P_{max\,III})$$

$$= \sin^{-1}(1/1.5)$$

$$= 0.729 \text{ rad}$$

$$\therefore \delta_{max} = \underline{\underline{2.411 \text{ rad}}}$$

$$\text{Area } A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - P_{e\,II}) \cdot d\delta$$

$$= \int_{\delta_0}^{\delta_{cr}} P_m \cdot d\delta$$

$$= P_m \left\{ \delta_{cr} - \delta_0 \right\}$$

$$= 1 \left\{ \delta_{cr} - 0.451 \right\}$$

$$\text{Area } A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{e\,III} - P_m) \cdot d\delta$$

$$= \int_{\delta_{cr}}^{\delta_{max}} P_{e\,III}\, d\delta - \int_{\delta_{cr}}^{\delta_{max}} P_m \cdot d\delta$$

$$= \int_{\delta_{cr}}^{\delta_{max}} P_{max\,III} \sin\delta \, d\delta - P_m \cdot \delta \Big\}_{\delta_{cr}}^{\delta_{max}}$$

$= \delta_{cr} - 0.451$

$\therefore A_1 = \delta_{cr} - 0.451$

$$= -P_{max\,II} \cos\delta \Big\}_{\delta_{cr}}^{\delta_{max}} - P_m \cdot \delta \Big\}_{\delta_{cr}}^{\delta_{max}}$$

$$= -1.5 \{ \cos\delta_{max} - \cos\delta_{cr} \} - \{ \delta_{max} - \delta_{cr} \}$$

$$= -1.5 \{ \cos 2.411 - \cos\delta_{cr} \} - \{ 2.411 - \delta_{cr} \}$$

$$A_2 = 1.117 + 1.5 \cos\delta_{cr} - 2.411 + \delta_{cr}$$

On Equating both Areas.

$$\cancel{\delta_{cr}} - 0.451 = 1.117 + 1.5 \cos\delta_{cr} - 2.411 + \cancel{\delta_{cr}}$$

$$1.5 \cos\delta_{cr} = 0.843$$

$$\delta_{cr} = \cos^{-1}\left(\frac{0.843}{1.5}\right)$$

$$= \underline{\underline{55.8^{\circ}}} = 0.562 \text{ radians}$$